Vector Calculus Formulas

Fundamental theorems (main result) Here, $\mathbf{F}(x, y, z) = P(x, y, z)\mathbf{i} + Q(x, y, z)\mathbf{j} + R(x, y, z)\mathbf{k}$.

FT of Line Integrals:	If $\mathbf{F} = \nabla f$, and the curve C has endpoints A and B, then $\int_C \mathbf{F} \cdot d\mathbf{r} = f(B) - f(A)$.
Green's Theorem:	$\iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA \;=\; \oint_C \mathbf{F} \cdot d\mathbf{r}$ (circulation-curl form)
Stokes' Theorem:	$\iint_S \nabla \times \mathbf{F} \cdot \mathbf{n}\, d\sigma \;=\; \oint_C \mathbf{F} \cdot d\mathbf{r}$, where C is the edge curve of S
Green's Theorem:	$\iint_D \nabla \cdot \mathbf{F}\, dA \;=\; \oint_C \mathbf{F} \cdot \mathbf{n}\, ds$ (flux-divergence form)
Divergence Theorem:	$\iiint_D \nabla \cdot \mathbf{F}\, dV \;=\; \oiint_S \mathbf{F} \cdot \mathbf{n}\, d\sigma$

Differential elements

Along a parametrized curve $\mathbf{r}(t)$, $t \in [a, b]$, we have $ds = \left|\frac{d\mathbf{r}}{dt}\right| dt$.

Along a parametrized surface $\mathbf{r}(u, v)$, $(u, v) \in D$, we have $d\sigma = \left|\frac{\partial \mathbf{r}}{\partial u} \times \frac{\partial \mathbf{r}}{\partial v}\right| du\, dv$.

Curl and divergence

For a continuously differentiable 3D vector field $\mathbf{F}(x, y, z) = P(x, y, z)\mathbf{i} + Q(x, y, z)\mathbf{j} + R(x, y, z)\mathbf{k}$,

$$\text{curl}\, \mathbf{F} \;=\; \nabla \times \mathbf{F} \;:=\; \left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right)\mathbf{i} + \left(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}\right)\mathbf{j} + \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)\mathbf{k}\,.$$

$$\text{div}\, \mathbf{F} \;=\; \nabla \cdot \mathbf{F} \;:=\; \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) \cdot (P, Q, R) \;=\; \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}\,.$$

Other 3D spatial coordinates

$$\begin{aligned}
r &= \sqrt{x^2 + y^2} = \rho \sin\phi, \\
\rho &= \sqrt{x^2 + y^2 + z^2}, \\
x &= r\cos\theta = \rho \sin\phi \cos\theta, \\
y &= r\sin\theta = \rho \sin\phi \sin\theta, \\
z &= \rho \cos\phi\,, \\
dV &= dz\, dy\, dx \\
&= r\, dz\, dr\, d\theta \\
&= \rho^2 \sin\phi\, d\rho\, d\phi\, d\theta\,.
\end{aligned}$$