1 Dfn. A directed graph $G$ is an ordered pair of sets $(V, E)$ such that $V$ is a nonempty, finite set of objects called vertices and $E$ is a set of ordered pairs of vertices.

2 Note. A directed graph differs from a graph only in that the edges have a “direction” given by the ordered pair. If $(v, w)$ is an edge of a directed graph, we draw the graph with an arrow that points from $v$ to $w$. For any two vertices $v$ and $w$, there are two possible edges between $v$ and $w$, the edges $(v, w)$ and $(w, v)$.

3 Dfn. A tournament is a directed graph $T$ such that for every pair $v, w$ of vertices, either $(v, w)$ or $(w, v)$ is an edge of $T$ but not both.

4 Note. Think of a tournament as the result of a round robin tournament played by teams represented by the vertices. An edge $(v, w)$ represents the fact that team $v$ defeated team $w$.

5 Dfn. If $(v, w)$ is an edge of a directed graph $G$, we say that $v$ dominates $w$.

6 Dfn. For any vertex $v$, the indegree of $v$, $d_i(v)$, is the number of vertices $w$ such that $w$ dominates $v$. The outdegree of $v$, $d_o(v)$, is the number of vertices dominated by $v$.

7 Question. How many nonisomorphic directed graphs are there with 3 vertices?

8 Question. How many nonisomorphic tournaments are there with 3 vertices?

9 Dfn. A path in a directed graph is a sequence of distinct vertices $v_0, \ldots, v_k$ in $V$ such that $v_{i-1}$ dominates $v_i$ for every $i$ such that $1 \leq i \leq k$.

10 Dfn. A Hamiltonian path in a directed graph is a path that contains every vertex of $G$.

11 Theorem. Every tournament contains a Hamiltonian path.

12 Dfn. A directed graph $G$ is transitive if whenever $(u, v)$ and $(v, w)$ are edges of $G$, then $(u, w)$ is an edge of $G$.

13 Theorem. If $T$ is transitive then there is a vertex $v$ such that $v$ dominates every other vertex in $T$.

14 Theorem. A tournament is transitive iff it contains exactly one Hamiltonian path.

15 Dfn. A cycle in a directed graph is a path for which the first and last vertices are the same.

16 Theorem. A tournament is transitive iff it contains no cycles.