September 27 — Soundness of $F_T$

1. Notation: If $X$ is a set of formulas and $S$ a sentence, $X \vdash_T S$ means that there is a proof in $F_T$ of $S$ from premises in $X$. $X \models_T S$ means that $S$ is a tautological consequence of the formulas in $X$.

2. A truth assignment is a function $h$ that assigns to every propositional variable a value in the set $\{\text{True}, \text{False}\}$. If $h$ is a truth assignment, we extend $h$ to a function $\hat{h}$ that maps all propositional formulas into the set $\{\text{True}, \text{False}\}$. The extension is by induction on the definition of the formula (“truth-tables”). So $X \models_T S$ means that for every truth-assignment $h$, if $\hat{h}(P) = \text{True}$ for all $P \in X$ then $\hat{h}(S) = \text{True}$.

3. The Soundness Theorem for propositional logic:

   **Theorem 1 (Soundness)** Suppose that $X$ is a set of formulas and $S$ a formula. If $X \vdash_T S$ then $X \models_T S$.

4. The proof is a proof by induction on the proof in $F_T$ of $S$. The book casts the proof as a proof by contradiction. That’s okay but a bit inelegant.

5. Let $S_1, S_2, \ldots, S_k$ be the lines of the proof of $S$. The key fact to prove is the following: For every $i \leq k$ if $Y$ is the set of premises for step $i$, then $Y \models S_i$.

6. The proof has boatloads of cases depending on which rule was used to prove $S_i$.

**Homework**

- Read LPL 8.3 and 17.1.
- Do LPL problems 8.41 and 8.42.