November 11 - Noncomputable languages

1. $K = \{ M \mid M(M) \downarrow \}$.
2. $K$ is not computable.
3. $K$ is computably enumerable.
4. Showing sets noncomputable:
   (a) $A$ is reducible to $B$ (written $A \leq_m B$) if there is a computable function $f$ such $\sigma \in A$ iff $f(\sigma) \in B$
   (b) If $A$ is reducible to $B$ and $B$ is computable then $A$ is computable.
   (c) $\text{TOT} = \{ M \mid \forall \sigma (M(\sigma) \downarrow) \}$
   (d) $\text{TOT}$ is not computable (since $K$ is reducible to $\text{TOT}$).
   (e) $\text{FIN} = \{ M \mid M(\sigma) \downarrow \text{ for only finitely many } \sigma \}$.
   (f) $\text{FIN}$ is not computable.
5. Let $\text{FOL-SAT} = \{ S \mid S \text{ is a satisfiable sentence of first-order logic} \}$ (here fix a first-order language $L$) Turing showed that $\text{FOL-SAT}$ is not computable by showing that $K$ is reducible to $\text{FOL-SAT}$. This shows that the Entscheidungsproblem is not decidable.

Homework

- Read Notes Section 4.6.
- Do problems 4.6.2,3. (Hint: in both cases, show $K$ is reducible to the given language.)
- Suppose that $A$ is a computably enumerable set. Show that $A \leq_m K_1$. 