Solutions to HWs #17.

Let \(N_1\) be a positive odd integer such that
\[
S_1 := 1 + \frac{1}{3} + \frac{1}{5} + \cdots + \frac{1}{N_1} > 3,
\]
and, as a result,
\[
S_2 := S_1 - \frac{1}{2} > 2.
\]
Iteratively, define \(N_k\) to be a positive odd integer such that
\[
S_{2k-1} := S_{2k-2} + \frac{1}{2+N_{k-1}} + \frac{1}{4+N_{k-1}} + \cdots + \frac{1}{N_k} > (2k-2) + 3 = 2k+1,
\]
and
\[
S_{2k} := S_{2k-1} - \frac{1}{2k} > S_{2k-1} - 1 > 2k.
\]
Then the series
\[
\left(1 + \frac{1}{3} + \frac{1}{5} + \cdots + \frac{1}{N_1}\right) - \frac{1}{2} + \left(\frac{1}{2+N_1} + \frac{1}{4+N_1} + \cdots + \frac{1}{N_2}\right) - \frac{1}{4} + \cdots
\]
is a rearrangement of the original (alternating harmonic) series. While we have that \(S_n \geq n\) for each \(n\), that is not the same as saying the entire sequence of partial sums of the rearranged series goes to \((+\infty)\). But that follows from the fact that the partial sums in between \(S_{2n-1}\) and \(S_{2n}\) are all larger than \(S_{2n-1} > 2n - 1\).