PROBLEM OF THE WEEK

Welcome back everyone! Below is the first problem of the new year.

I tried to find out over the summer just how long we have had a problem of the week. It appears to be at least 25 years, but no one is quite sure. Since I couldn’t even determine how long it has been going, there was no way for me to figure out how many problems there have been, but it seems quite certain to be well over 500. You can join a long tradition of Calvin students by submitting a solution to a problem.

I’ve decided to start numbering problems consecutively beginning in January, 2000. That makes this problem 37 (at least approximately) of the 2000’s.

I encountered this problem at MathFest this summer. This result was used to perform several mathematical card tricks. No proof was given in the talk, so I played around with it myself. I hope you enjoy it, too.

37. Consider a finite sequence of distinct integers. A subsequence is a sequence formed by deleting some items from the original sequence without disturbing their relative ordering. A subsequence is called monotone if it is either increasing (each term is larger than the one before it) or decreasing (each term is smaller than the one before it). For example, if the sequence is 4, 6, 3, 5, 7, 1, 2, 9, 8, 10, then 4, 6, 8, 10 is a monotone (increasing) subsequence of length 4 and 6, 5, 2 is a monotone (decreasing) subsequence of length 3.

a) Find a sequence of 9 distinct integers that has no monotone subsequence of length 4.
b) Show that every such sequence of length 10 has a monotone subsequence of length 4.
c) Generalize this result. (How long must the sequence be to guarantee a monotone subsequence of length $n$?)

How the Problem of the Week works:

1. ANY CALVIN STUDENT is invited to participate in the Problem of the Week on any week. Solutions (or partial solutions) may be submitted by individual students or by groups of students.

2. COPIES of the Problem of the Week will be hung on the bulletin board outside the Department office and in various locations around the Department of Mathematics and Statistics. Additional copies are available in one of the boxes outside the office and on the web at http://www.calvin.edu/~rpruim/pow/

3. SOLUTIONS to this problem are due on Thursday, September 13. Solutions should be turned in to Professor Pruim (NH 284). Be sure to include your name(s) on your paper.

4. A LIST OF SOLVERS AND EXAMPLE SOLUTIONS will be posted on the bulletin board outside the Mathematics Department office.