PROOF. Let \( n \in J \), and let \( P(n) \) be the statement \( 1 + 2 + \ldots + n = \frac{n(n+1)}{2} \).

We will use induction to show that \( P(n) \) is true for all \( n \in J \).

(Base case): Suppose \( n = 1 \). \( 1 = \frac{1(1+1)}{2} = 1 \) so \( P(1) \) is true.

(Inductive step): Suppose that \( P(k) \) is true. We must show that \( P(k+1) \) is also true. \( 1 + 2 + 3 + \ldots + k = \frac{k(k+1)}{2} \) since \( P(k) \) is true.

So \( 1 + 2 + 3 + \ldots + k + k + 1 = \frac{k(k+1)}{2} + k + 1 \)

\[
= \frac{k(k+1) + 2(k+1)}{2}
\]

\[
= \frac{(k+1)(k+2)}{2}
\]

This last equality asserts that \( P(k+1) \) is true. By the principle of induction \( P(n) \) is true for all \( n \in J \). \( \square \)