Below are some problems for you to work on. Work in groups of two or three. Use *Mathematica* to help with some of the technical details (like computing derivatives, performing algebra, graphing, etc.). In each of your answers be sure to **explain your reasoning** very carefully. The point is for you to learn how to **justify the claims you make**. Give exact values whenever possible, decimal approximations otherwise.

If you want to produce your report within *Mathematica* you may do so, but you must include full sentence explanations. Use Format—Style—Text to format your text cells so that they look like text rather than like *Mathematica* commands.

The first few problems focus on the relationships between functions and their first and second derivatives.

1. Let \( f(x) = 5 - 4x - 2x^2 + 4x^3 - x^4 \). Locate all critical points and inflection points and determine which critical points are local and absolute minimum and maximum values. (Remember to explain your reasoning.)

2. Find the absolute extrema of the given functions on the indicated interval.
   a) \( f(x) = \cos 2x + 2 \cos x \) on \( 0 \leq x \leq \pi \)
   b) \( f(x) = 2 \sin x - \cos 2x \) on \( 0 \leq x \leq 2\pi \)

3. Show that the sum of a positive number and its reciprocal must be at least 2.

4. Consider the family of functions defined by \( f(x) = \frac{c x^2}{1 + x^2} \) where \( c \) is a constant. Graph several members of the family and investigate how the minimum and maximum points and inflection points move as \( c \) is changed.

   **Mathematica** Notes:
   
   (a) Be sure to put either a space or an asterisk (*) between the \( c \) and the \( x \) to indicate multiplication. Otherwise, *Mathematica* will treat \( cx \) as a single variable.

   (b) You can get *Mathematica* to make lists for you using the `Table[]` command. In order to use it within the `Plot[]` command you must wrap this inside an `Evaluate[]`. Here are two examples:

   ```plaintext
   Table[x^2+k, {k,0,10,2}]
   Plot[Evaluate[Table[x^2+k, {k,0,5}]], {x,-4,4}]
   ```

   This last problem is related to the Mean Value Theorem.

5. A quadratic function is a function of the form \( f(x) = Ax^2 + Bx + C \) where \( A \neq 0 \).
   a) Find a quadratic function \( f(x) \) satisfying: \( f(0) = f'(0) = f''(0) = 2 \).
   b) Suppose that \( f(x) = Ax^2 + Bx + C \) has roots \( r \) and \( s \), (i.e. \( f(r) = f(s) = 0 \)). Show that \( f'(c) = 0 \) where \( c \) is the midpoint between \( r \) and \( s \).
   c) Show that for any quadratic function \( f(x) = Ax^2 + Bx + C \), the \( x \)-value \( c \), guaranteed by the Mean Value Theorem applied to \( f(x) \) on \([a, b]\) is always the midpoint between \( a \) and \( b \).