2018 Workshop in Geometric Topology

Problems

Greg Friedman: Given a map $\phi : X \to M_n(\mathbb{C})$ (the set of $n \times n$ matrices with complex entries) such that the eigenvalues of each $\phi(x)$ are distinct, consider the fiber bundle over X in which the fiber over each $x \in X$ is $\mathbb{C}^n$ is expressed as the direct sum of the 1-complex dimensional eigenspaces of $\phi(x)$. This bundle can be thought of as a *braided complex line bundle.*

Question: Is there a name of these objects and are there known analogues of characteristic classes for them?

Craig Guilbault: *Question:* Does the Mazur compact contractible 4-manifold contain a pair of disjoint of disjoint spines?

Bob Daverman: Let X be a subset of the n-sphere S^n.

Question: When can a cell-like map $f : S^n \to S^n$ such that $f|X = id$ be approximated by a homeomorphism $h : S^n \to S^n$ such that $h|X = id$?

Remark: The answer is known to be "yes" if X is a knot in S^3.

Mike Mihalik: *Question 1:* Can the Pontryagin sphere be the boundary of a hyperbolic or CAT(0) group?

Remark: After the Problem Session, several participants observed that the Davis construction applied to the cone over of compact orientable surface of positive genus can yield both hyperbolic and CAT(0) groups whose boundaries are Pontryagin spheres. Also, this construction is implicit in the methods of Dranishnikov's article "Boundaries of Coxeter groups and simplicial complexes with given links", *Journal of Pure and Applied Algebra* **137** (1999),139-151.

Question 2: If G is a word hyperbolic group which is simply connected at ∞, must ∂G be simply connected?

Remark: The answer to Question 2 is "no" for CAT(0) groups. For example, if CK is the Croke-Kleiner group $< w, x, y, z \mid [w,x] = [x,y] = [y,z] = 1 >$, then $CK \times \mathbb{Z}$ is simply connected at ∞, but none of its boundaries are simply connected.

Nathan Sunukjian: *Fact:* Every knot $S^1 \subset S^3$ can be unknotted by a sequence of crossing changes. Stated another way: every knot in S^3 can be unknotted by a sequence of ±1 surgeries on loops encircling a crossing.

Question 1: Can every 2-knot $S^2 \subset S^4$ be unknotted by a sequence of log transforms on tori ($T^2 \times D^2$ ↺ in S^4)?

Question 2: Is this true for knotted tori T^2 in S^4?

Ric Ancel: *Definition:* A contractible open n-manifold is *splittable (in the sense of Gabai)* if it can be written as a union $U \cup V$ of open sets U and V such that U, V and $U \cap V$ are homeomorphic to $\mathbb{R}^n$.

Question: Is the interior of the Mazur compact contractible 4-manifold splittable?

Background:

1) D. Gabai (2011) showed that the Whitehead contractible open 3-manifold is splittable.
2) D. Garity, D. Repovs and D. Wright (2018) showed that there exist uncountably many non-homeomorphic splittable contractible open 3-manifolds and there exist uncountably many non-homeomorphic non-splittable contractible open 3-manifolds.
3) Pete Sparks (2015) showed there exist uncountably many non-homeomorphic splittable contractible open 4-manifolds.
4) Let $n \geq 5$. The interior of every compact contractible n-manifold is splittable, and every n-dimensional Davis manifold is splittable. Furthermore, every contractible open n-manifold can be written as a union $U \cup V$ of open sets U and V such that U and V are homeomorphic to $\mathbb{R}^n$ and $U \cap V$ is contractible. No example of a non-splittable contractible open n-manifold is known.
5) Sparks and Ancel (2018) showed that the Dunce Hat spine of the Mazur 4-manifold is *not* splittable in the sense that it can't be expressed as the union of two compact contractible proper subpolyhedra.

Remark. If a spine of the Mazur 4-manifold were splittable, then the interior of the Mazur 4-manifold would also be splittable. However, the converse doesn't hold: the non-splittability of a spine of a compact manifold does not imply the non-splittability of the interior of the manifold. Indeed, the Dunce Hat can be embedded in 3-space where it is the spine of a 3-ball; the Dunce Hat is non-splittable but the interior of a 3-ball is splittable. So the result stated in 5) doesn't resolve the Question.

Dubravko Ivansic: *Question and, by acclamation of groans, worst math pun of the Workshop:* How many sides does a *Muskegon*[1] have?

[1] Accent on first syllable.