Deficient and Multiple Points of Maps into Manifolds

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Abstract. For a map $f : M \to N$ between orientable manifolds of same dimension, a point $y \in N$ is called (*essentially*) *deficient* if $f^{-1}(y)$ has less then $|\deg f|$ (essential) points. Such points were studied by several topologist, e.g. Hopf, Honkapohja, Church, Timourian and Walsh.

The following result was proved by Church and Timourian.

Suppose M and N are connected orientable n-manifolds and $f : M \to N$ is a proper map with degree $|\deg f| \neq 0$. Let E_f be the set of essentially deficient points of f.

(1) *Then $\dim E_f \leq n-1$ and, moreover, E_f contains no closed (in N) subset of dimension $n-1$.*

(2) *If f is discrete, then $\dim(\overline{E}_f) \leq n-2$.*

They also generalized this theorem to manifolds which are not necessarily orientable replacing $|\deg f|$ by Hopf's absolute degree $A(f)$.

We extend the notions of absolute degree and essentially deficient points with respect to maps from some spaces (in particular from CW-complexes) to manifolds. Then we prove a result, which extends the result of Church and Timourian, where domain is a space which satisfies certain cohomological property.

We consider also a related notion of *multiple* point of a map. A point $x \in X$ is a multiple point of a map $f : X \to Y$ if $f^{-1}(f(x)) \neq \{x\}$. We study density of multiple points in X, and also provide examples where its complement is dense.

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