New Symmetries of Stable Homotopy Groups

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Abstract. This will really be a talk about abstract algebra and some very elementary number theory, but the motivation for the work comes from topology. Work of Bousfield, Ravenel, Morava, Miller, Wilson, and others during the 1970s and 1980s established that the stable homotopy groups of spheres can be decomposed into many periodic families, each of which repeats every $2(p^n - 1)$ dimensions, where p is a prime and n is a nonnegative integer; and furthermore, if one fixes p and n, then there are spectral sequences that one can, in principle, use to calculate the $2(p^n - 1)$-periodic families in the stable homotopy groups of spheres. These spectral sequence calculations are extremely difficult, however, and complete calculations have only been made for $n < 3$.

When $p > n + 1$, these spectral sequence calculations begin with the cohomology of a certain differential graded algebra defined by Ravenel. In this talk, we give a very explicit description of this (surprisingly simple!) differential graded algebra, and we explore the problem of finding symmetries (i.e., automorphisms) of this differential graded algebra for large n. We show how we reduce the problem of finding such symmetries to a very explicit and elementary problem in number theory, and we demonstrate some new symmetries–which give rise to new operations on the $2(p^n-1)$-periodic stable homotopy groups of certain CW-complexes called Smith-Toda complexes–which our elementary, explicit approach produces. (The focus of the talk will be on the very approachable algebra and elementary number theory which we used to get our new results; the ideas from homotopy theory are there to motivate our work, but our talk should be understandable to an audience which does not know any homotopy theory!)