Conditions under which a sewing of crumpled $n$-cubes yields $S^n$

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Abstract. A crumpled $n$-cube is the closure of the complement of an $n$-cell wildly embedded in $S^n$, and a sewing of crumpled $n$-cubes is a homeomorphism between their boundaries. Associated with any such sewing $h : \text{Bd } C_1 \to \text{Bd } C_2$ is the sewing space $C_1 \cup_h C_2$, namely the quotient obtained from the disjoint union of $C_1, C_2$ under identification of each $x \in \text{Bd } C_1$ with $h(x) \in \text{Bd } C_2$. It is known that $C_1 \cup_h C_2$ is an $n$-manifold (and, hence, $S^n$) if it satisfies the following Strong Mismatch Property: any two maps $f_i : I^2 \to C_i$ can be approximated, arbitrarily closely, by maps $F_i : I^2 \to C_i$ such that $F_2(I^2) \cap h(F_1(I^2) \cap \text{Bd } C_1) = \emptyset$. A related Weak Mismatch Property, which is known to be a necessary condition for $C_1 \cup_h C_2$ to be a manifold when $n \geq 5$, allows the approximations $F_i$ to range into $S^n$ instead of being restricted to $C_i$. Theorem: In case $C_1, C_2$ are crumpled $n$-cubes satisfying the Disjoint Disks Property, then the Weak Mismatch Property is a necessary and sufficient condition for $C_1 \cup C_2$ to be $S^n$. Also discussed will be an Intermediate Property and sharp conditions under which a sewing with this property yields $S^n$. 