PROBLEM SESSION

1. **Lawrence Brenton**
   (a) Let $X$ be the cone on a homology 3-sphere $M$. Does there exist a Lorentzian metric $g$ on $X$ that is homogeneous on cross sections such that $(X, g)$ satisfies the dominant energy condition?
   (b) If “no,” where does the obstruction lie?
   (c) Will the spacetimes of part (a) always recollapse in a “big crunch,” or does this depend on the choice of metric?

2. **Robert Daverman**
   (a) If $X$ is a compact ANR homology 3-manifold, does there exist a real 3-manifold $M$ such that $M$ is homotopy equivalent to $X$?
   (b) If so, does $X$ embed in $M \times \mathbb{R}$?
   (c) If so, is $X \times \mathbb{R} \cong M \times \mathbb{R}$?

3. **David Wright**
   Are there examples of compact 3-manifolds (or $n$-manifolds) in which every homeomorphism is isotopic to the identity?

4. **Tadek Dobrowolski**
   Let $X$ be a contractible, locally contractible compact metric space. Does $X$ have the fixed point property?
   The answer is known to be “yes” if there exists a function $\lambda : X \times X \times [0, 1] \to X$ such that
   \[
   \lambda(x, y, 0) = x,
   \lambda(x, y, 1) = y, \text{ and }
   \lambda(x, x, t) = x \text{ for } 0 \leq t \leq 1.
   \]
   Every AR has such a function.

5. **Steve Ferry**
   Is there a sequence of Riemannian manifolds, sharing a fixed contractibility function, that approach (in Gromov-Hausdorff space) an infinite dimensional space with a bound on volume?
   Definitions: A \textit{contractibility function} on $M$ is a function $\rho : (0, \infty) \to (0, \infty)$ such that for every $t > 0$ and for every $x \in M$
the ball of radius \( t \) in \( M \) centered at \( x \) is contractible in the ball of radius \( \rho(t) \). If \( X \) and \( Y \) are compact metric spaces, the Gromov-Hausdorff distance \( d_{\text{GH}}(X, Y) \) is defined by

\[
d_{\text{GH}}(X, Y) = \inf \left\{ d^Z(X, Y) \mid Z \text{ metric space } \supset X, Y \right\},
\]

where \( d^Z \) is the usual Hausdorff distance between subcompacta of \( Z \).

6. **Craig Guilbault**

Given a homomorphism \( \mu : G \to \pi_1(M) \), with \( G \) a finitely generated group and \( M \) a closed manifold, such that \( \ker(\mu) \) is perfect, does there exist a 1-sided \( h \)-cobordism that realizes \( \mu \)? In other words, does there exist a triple \((W, M, M^*)\) of manifolds such that \( \partial W = M \sqcup M^* \), \( M \hookrightarrow W \) is a homotopy equivalence, and

\[
\begin{array}{ccc}
\pi_1(M^*) & \longrightarrow & \pi_1(W) \\
\cong & \uparrow & \cong \\
G & \xrightarrow{\mu} & \pi_1(M)
\end{array}
\]

commutes? [This is the reverse of Quillen’s +-construction.]

7. **Sasha Dranishnikov**

(a) Is \( \text{asdim}(X) = \dim(\nu X) \)?
(b) If \( \Gamma \) is a CAT(0) group, is \( \text{asdim}(\Gamma) < \infty \)?
(c) For \( n \geq 2 \), does there exist a Coxeter group \( \Gamma \) such that \( \text{vcd}_Q \Gamma = 2 \) and \( \text{vcd}_Z \Gamma = n \)?