CHAPTER 3

Advanced techniques in GSP

This chapter introduces two features of GSP that greatly enhance its usefulness. Custom tools allow you to quickly accomplish routine constructions and action buttons allow you to make sketches that illustrate a process, not just a finished product.

3.1 CUSTOM TOOLS

One of the most effective ways to tap the power of GSP is to create custom tools to perform constructions that must be repeated many times. Once you learn how to make custom tools of your own, you should begin to accumulate a collection of specialized tools that you can use when you need them.

To create a custom tool, you begin by making a GSP sketch that includes the construction you want to make into a tool. Once you have made the sketch, you use the selection arrow to select the the objects in the construction and then choose Create New Tool under the Custom Tool icon in the tool bar. You must be careful to select a set of objects that are related in such a way that at least one of them is completely determined by the others. The objects you selected that are to be produced by the tool are called the *results* and the objects that don't depend on any others are called *givens*. Any object that is part of the construction but is neither a given nor a result is called an *intermediate object*.

Here is a specific example. Suppose we wish to create a tool that constructs the perpendicular bisector of a segment. The first step is to construct two points A and B, the segment $\overline{AB}$ joining them, the midpoint M of the segment, and finally the line ℓ that is perpendicular to $\overline{AB}$ and passes through M. The perpendicular bisector of a segment is completely determined by the endpoints of the segment, so you can create the tool by selecting the two points A and B and the line ℓ. Choose Create New Tool and a dialog box will pop up. Type in the name you want to give your tool and click OK. Now your tool is ready to use and its name should appear in the menu that pops up when you press on the Custom Tool icon. To use it, simply choose it by name from the pop-up menu and then click in your sketch. The first time you click, the point A should be constructed; the second time you click, both the point B and the line ℓ will be constructed.

In the example above, the points A and B are the givens and the line ℓ is the result. The segment $\overline{AB}$ and the midpoint M are intermediate objects. When you select the objects that are to be part of your tool, you can control which of them are to be shown in the result and which are to be hidden. It is usually best to make tools that have only points as givens, but it is also possible to use other objects as the givens. For example, you could use the segment $\overline{AB}$ as the given in your perpendicular bisector tool instead of the endpoints A and B. To accomplish this, you simply select the segment $\overline{AB}$ and the line ℓ when you create the tool. On the other hand, you could include the segment $\overline{AB}$ and midpoint M as results. To accomplish that, you should select all of the objects you have constructed (the points A and B, the segment $\overline{AB}$, the midpoint M and the line ℓ) when you create the tool. If you make the tool that way, the givens are A and B since they are the parents of everything else in the construction.

EXERCISES

***3.1.1.** Create a Perpendicular Bisector tool exactly as in the example above. Your tool should accept two points as givens and should produce only the perpendicular bisector as result.

***3.1.2.** Create a second version, Perpendicular Bisector 2, of the perpendicular bisector tool. This one should again have the two points as givens, but should produce the segment, the midpoint, and the perpendicular bisector as results.

***3.1.3.** Make a third version of the perpendicular bisector tool that accepts a segment as the only given and produces both the midpoint and the perpendicular bisector as results.

Any tool you make becomes part of the sketch in which it is created. The tool will be available for use any time that particular sketch is open. The Sketchpad directory that is installed on your computer includes a Tool Folder. Any custom tool that is part of a document stored in this folder is automatically loaded when you start Sketchpad. It is best to put your tools in that folder, if possible, so that they are always available for use when you need them. The alternative is to open the sketch containing the tool and leave it open as long as you might want to use the tool; there is no limit to the number of Sketchpad documents you can have open at one time. It is usually best to include each tool in a separate Sketchpad document. The sketch and the custom tool contained in the sketch can have the same name.

If you choose Show Script View from the Custom Tool pop-up menu you will see a written description of the tool. It is sometimes helpful to look at that window in order to determine the object requirements of a given tool. You might also want to consult the script view if you are trying to determine what a tool does and how it was originally defined. There is a box at the top of the window where you can type comments; it is a good idea to include comments, both for your own later use and for the benefit of anyone else who might use the tool. You can also include explanatory text in the sketch itself.

EXERCISES

***3.1.4.** Make a custom tool that accepts two points as givens and whose result is an equilateral triangle with the given points as the endpoints of one side. Save your tool for future use. [Compare Exercise 1.3.3.]

***3.1.5.** Make a custom tool that accepts two points as givens and whose result is a square with the given points as the endpoints of one side. Save your tool for future use. [Compare Exercise 1.3.4.]

***3.1.6.** Make a custom tool that accepts a segment as given and whose result is a square region with the given segment as one side. Save your tool for future use.

***3.1.7.** Create a tool that accepts a segment as its given and constructs a circle with the given segment as diameter. Save your tool for later use.

3.2 ACTION BUTTONS

As the name suggests, action buttons allow you to add action to your GSP sketches. They also allow you to make more interesting presentations with GSP. In this section you will learn how to create action buttons; for the remainder of the course you should make use of them to improve your GSP sketches.

To create an action button, use the selection arrow to select some objects and then choose the appropriate type of action button from the Action Buttons submenu that is found under the Edit menu in GSP. The action button itself is considered to be a GSP object and the objects you selected when you created the button are its parents.

Here is a description of the various kinds of action buttons.

- Hide/Show. This button allows you to alternately hide or show an object or collection of objects. To use it, you simply select the objects you want to hide or show and then choose Hide/Show from the action button submenu. A button will appear in your document. When you click on it with the selection arrow, the objects will disappear; when you click on it a second time they will reappear.
- Animation. You can add a button that turns animation on or off. Simply select the objects you want to animate and then choose Animation from action button submenu. A window will appear that can can be used to control some of the characteristics of the animation if you wish. Click OK to make the window go away. An animate button will appear in the sketch. The first time you click on it with the selection arrow the animation will be turned on and the next time you click on the button the animation will be turned off.
- Movement. To use this button, select a pair of points in your sketch and then choose Movement in the action buttons submenu. Again a window will appear that can can be used to control some of the characteristics of the movement if you wish. Click OK to make the window go away. A button will appear in your sketch. When you click on it, the first point will move to the second. You also have the option of selecting several pairs of points at once;

when you do so, the Movement button will cause the first point in each pair to move to the second point in the pair.

- Presentation. A presentation button automatically activates a group of other buttons. The buttons can be activated either simultaneously or in sequence. Use a presentation button to choreograph a complex set of motions or to present a Sketchpad slide show. To create a presentation button, select a group of buttons by clicking on the black rectangle at the left edge of the buttons. Then choose Presentation under the action button submenu. A window will pop up in which you can control some of the characteristics of the presentation. Click OK to make the window go away.
- Link. A Link button allows you to link to another page of your document or to an internet URL.
- Scroll A scroll button scrolls the sketch window so that a selected point is located either in the center of the window or at the top left corner of the window. To create a scroll button, select a point and then choose Scroll under the action button submenu. Use a scroll button in large sketches to position the window or to move to a different part of your sketch.

3.3 THE EULER LINE REVISITED

The sketches you created in the last chapter can be greatly improved with the use of custom tools and action buttons.

EXERCISES

***3.3.1.** Make a GSP tool that constructs the centroid of a triangle. Your tool should accept three vertices of a triangle as the givens and produce the triangle and the centroid G as results. The tool should label the centroid G. The tool should not display the intermediate objects that were used in the construction of G.
[Hint: As you are constructing the centroid and before you create the custom tool, choose the Text tool and double click on the centroid. A Properties box will pop up. Type the label G in the box and click on Use Label in Custom Tools. Then proceed to create the tool. It will consistently name the centroid G.]

***3.3.2.** Make a GSP tool that constructs the orthocenter of a triangle. Your tool should accept three vertices of a triangle as givens and produce the triangle and the orthocenter H as results. The tool should label the orthocenter H. The tool should not display the intermediate objects that were used in the construction of H.

***3.3.3.** Make a GSP tool that constructs the circumcenter of a triangle. Your tool should accept three vertices of a triangle as givens and produce the triangle and the circumcenter O as results.

***3.3.4.** Make a new sketch that illustrates the Euler Line Theorem. First construct a triangle and then use the tools from the preceding exercises to create the three triangle centers. Hide any intermediate objects so that only the triangle itself and the three triangle centers are visible. Construct a line through two of the points and observe that it also passes through the third. Add hide/show buttons for the triangle, the three centers, and the Euler line. Add a presentation button that turns the sketch into a slide show that illustrates the Euler Line Theorem. Make sure that you have made good use of color and text boxes to make your sketch user friendly.

3.4 THE PYTHAGOREAN THEOREM REVISITED

The Pythagorean theorem is part of elementary Euclidean geometry, but it is worth studying again because of its importance to geometry generally. In the exercises below you will first explore the statement of the theorem and then look at Euclid's proof. Even though there are literally hundreds of different proofs of the Pythagorean theorem, Euclid's proof remains one of the most beautiful and elegant.

EXERCISES

***3.4.1.** Construct a right triangle. Use your tool from Exercise 3.1.6 to construct the square region on each side of the triangle. Measure the areas of the squares and verify the Pythagorean relationship.
[Hint: Be careful about the order in which the givens in your square tool are selected so that all three of the squares appear on the outside of the triangle.]

***3.4.2.** Construct a right triangle. Use your tool from Exercise 3.1.4 to construct an equilateral triangle on each side of the right triangle. Measure the areas of the associated triangular regions. Find a relationship between the areas.

***3.4.3.** Construct a right triangle. Use the tool from Exercise 3.1.7 to construct three circles whose diameters are the three sides of the triangle. (These circles will overlap.) Measure the areas of the circles and find a relationship between them. Explain how this relationship can be used to determine which is larger: a large pizza or the combination of one small and one medium pizza.[1]

3.4.4. State a theorem that summarizes the results of the last three exercises.
[Hint: Check Euclid's Proposition VI.31, if necessary.]

3.4.5. Find Euclid's proof of the Pythagorean theorem on the world wide web. Make sure you understand the proof.
[Hint: The Pythagorean theorem is Euclid's Proposition I.47. You can also find the proof on page 201 of [16]. The proof goes by the name of *bride's chair* for historical reasons that are quite obscure.]

***3.4.6.** Create a GSP sketch that illustrates Euclid's proof. The sketch should include action buttons that show objects as they are needed. It should also have buttons that show how triangular regions are related. There are two kinds of relationships: the first is a shear in which one vertex of a triangle moves along a line that is parallel to the base of the triangle and the second type of movement is a rotation about a point.

***3.4.7.** **Challenge Exercise.** Make a GSP sketch that animates Euclid's proof. Your sketch should show the triangles moving, first by a shear, then by a rotation, and finally by a second shear.

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[1] This is the origin of the familiar saying, "The pizza on the hypotenuse is equal to the sum of the pizzas on the legs."