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Multiscale Numerical Algorithms for Weather Forecasting and Climate Modeling: Challenges and Controversies

By John P. Boyd

Meteorological oracles come in three flavors:

- General Circulation Models
- Global Weather Forecasting Models
- Limited Area Models

General circulation models (GCMs) are run at low resolution (compared to forecasting codes) for long times (many model years, and months of wall-clock time) from arbitrary initial conditions to reach a statistical equilibrium, i.e., the climate. Global weather forecasting models (GWFMs) are run at high resolution for five to ten days of model time from observed initial conditions to make deterministic forecasts. Limited area models (LAMs) are "macroadaptive": A high-resolution grid over a hurricane or a continent is embedded, explicitly or implicitly, in a global model of lower resolution. The spiraling-out-of-control complexity of these codes is forcing a convergence of species; the British Met Office Unified Model, for example, is run as both a global and a limited area model.

Each model has a "dynamical core," which is the fluid mechanics, and the "physics," which is a very misleading name for everything else: a slew of subroutines for radiative transfer, photochemistry, air–sea interaction, boundary layer turbulence, the hydrologic cycle, cloud physics, and biogeochemical cycles too complicated to enumerate here. Consequently, GCMs now exceed a million(!) lines of code. These many processes are violently nonlinear and strongly coupled. A GCM or GWFM is the computational embodiment of a bar fight in a John Wayne western. The code cowboys who wrangle these megacodes are acutely aware that they are herding not sheep, but rather a flock of subroutines, ornery and independent as a herd of professors.

Mathematics department missionaries have often sallied forth to mathematize the meteorological primitives and spread the gospel of new algorithms. They are invariably surprised when the ungrateful savages tie them to a stake. As the bonfire is lit, they wail, "But it works for Burgers' equation!"

Meteorologists don't give a darn what works for Burgers' equation. Real atmospheric models suffer from Brittle Bone Syndrome: With so many interacting nonlinear feedbacks, atmospheric models "break if you breathe on them," to quote Mark Cane of Columbia. Clever adaptive algorithms that work for smooth, straight shocks disintegrate into computational anarchy when flayed by gravity waves, assaulted by moist convective instability, battered by highly temperature-sensitive photochemistry, and coupled to the vastly different time and space scales of the ocean.

Former SIAM board chair Werner Rheinboldt described applied mathematics as "mathematics plus." The description is doubly true in geophysics, where the "plus" is a thorough understanding of the intricate phenomenology of the ocean–atmosphere system.

Adaptive mesh refinement (AMR) is a good illustration. Can "microadaptation," which is a feature-following fine grid of complex topology, succeed?

Christiane Jablonowski of the University of Michigan developed a massively parallel, mass-and-energy-conserving microadaptive finite volume model that she has applied to an idealized case: a Rossby wave excited by uniform flow deflected by a mountain poking up from an otherwise flat planet at 45 degrees North. Refracted by variations in the Coriolis force, the wave curves into the southern hemisphere, then north across the equator again. The good news is that the fine grid successfully follows the twists and turns as the small length scales of the mountain are propagated to the other side of the world. The bad news is that earth does not have one mountain—it has a whole lot of mountains! A many-mountain world with microadaptation will have a grid that looks as convoluted as a finger painting by an enthusiastic preschooler.

Hence, the more spiritual code cowboys murmur the Adaptive Mesh Refiner's Prayer:

Please, God, No Waves!

The "physics" is also perilous for AMR. For example, condensation begins at a relative humidity q = 100%. As illustrated in Figure 1, however, a hill will force unsaturated air to rise, expand adiabatically, cool, and saturate, even

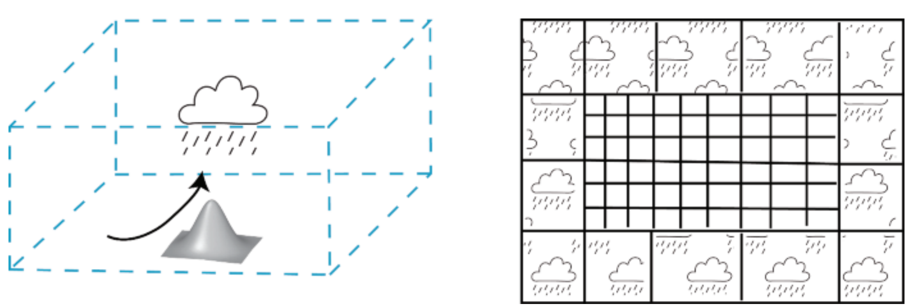

Figure 1. *Left: orographically forced sub-grid-scale thunderstorm. Right: poor parameterization of sub-grid-scale moist convection leads to rain in the coarse grid, but not in the fine grid, where the dynamics is most intense.*

though the average relative humidity in the box is less than 100%. To allow for such sub-grid-scale orographically forced thunderstorms, model convection must begin when $q < 100\%$. Logically, the threshold $q_t(h)$ should increase as the grid size h shrinks. Such scale-dependent parameterizations sometimes go awry, generating model rain in coarse grid boxes, but not in the fine grid cells where the convective threshold is higher—and where, if the adaptation has been done well, the convection should be most intense. Many processes must be similarly parameterized, inspiring the Geophysical AMR Prayer to St. Jude:

Deliver Us from the Physics!

St. Jude is the patron of Desperate Cases. But geophysical modeling is always desperate.

Another major challenge is the following bit of *mokita*, a term used in Papua New Guinea for "something that we all know but agree not to talk about."

Logarithmic Law of Arithmurgy
Insight grows logarithmically with the number of floating-point operations.

Extended Moore's Law
Computational power and scientific data, as measured in operations and bytes, both grow exponentially with time.

Corollary: Linearity of Progress with Time
Knowledge, as opposed to mere facts or data, grows linearly with time. (See Figure 2.)

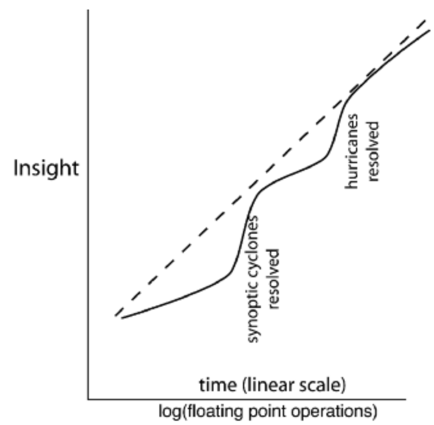

Figure 2. *In the long term, knowledge grows logarithmically with the number of floating-point operations (dashed curve), which is growth that is linear with time. The unaveraged (solid) curve is more like a flight of stairs because of jumps in insight whenever the grid spacing becomes sufficiently fine to resolve a new phenomenon or length scale.*

We have dignified these assertions with the rare word "arithmurgy"* to disguise the fact that what these laws describe is only an observed trend. Our other senses are logarithmic too, however. The electron microscope has blessed biology, but even the ability to see specimens at 100,000× magnification has hardly made biologists 100,000 times wiser than Pliny. The Hubble telescope has enormous light-gathering capacity, but modern astronomers are not a billion times more knowledgeable than Kepler. Similarly, our senses of hearing, vision, and touch operate logarithmically according to the Weber–Fechner law of physiology; the familiar decibel scale for sound is logarithmic.

Using the system of three ordinary differential equations that now bears his name, the late Ed Lorenz proved that neighboring trajectories diverge exponentially with time. Similar behavior in real forecasting codes implies that an exponential improvement in initial conditions, number of grid points, and so on extends the window of useful forecasting only linearly, as illustrated in [1]. Thus, the Logarithmic Law of Arithmurgy is rigorously true for weather prediction. Current forecasts are useful for about five days, but it is theoretically impossible to extend the window more than two weeks into the future, if that. There is still room for progress, however: in doubling tornado warning times, narrowing hurricane tracks, and extending global forecasts another day or two. Even so, weather and climate are fields of incremental improvements rather than dramatic breakthroughs, such as the discovery of buckyballs.

Climate modeling has its own limits. When different GCMs are compared, and/or when the same GCM is run with different parameters within the range of known uncertainties, the result is not a single number but rather a probability distribution. As shown in Figure 3, the most likely response to a doubling of greenhouse gases is a temperature rise of about three degrees C. Because temperature fell about six degrees during the last Ice Age, as reliably determined from sediment cores and other geological data, this would be a sort of half anti-Ice Age. The big worry is that there is a small but nonzero probability of huge change. With an expected rise in sea level by 2100 of only about three feet, Al Gore's twenty-foot flood in *An Inconvenient Truth* is such an outlier.

Gerard Roe and Marcia Baker [2] have argued convincingly that the nonlinear feedbacks, mostly positive and reinforcing, will never be known sufficiently well to narrow the probability distribution any time soon. The mokita of climate is that, irrespective of their theory, the probability distribution has in fact not narrowed significantly over the past decade.

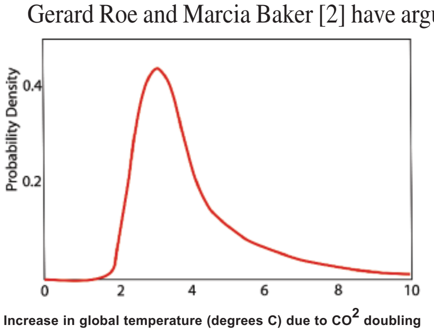

Figure 3. *Schematic of the probability density of general circulation models.*

Is climate prediction hopeless? Not quite. As Philip Thompson said in 1976, explaining his decision to become the first head of the Climate Project at the National Center for Atmospheric Research:

> "We can't forecast weather beyond five days. So, now let's do something really hard: predict climate decades into the future! Even so, it's worth a try because the potential payoff is so huge."

His words are still wise. Roe and Baker note that in a decade or two, we will be able to compare observed changes with forecasts, at long last narrow the probability distribution, and make a "midcourse correction" to climate policy. Despite the breathless media stories that see the bloody

*"Arithmurgy" is "number-working," from the Greek αριθμοσ, "number," and –εργοσ, "working."

hand of global warming in every hurricane, flood, and drought, the present day signs of climate change are too mild to narrow the probability distribution.

These three challenges are exemplars of a far longer list of topics in which mathematics and meteorology commingle. The National Science Foundation's Collaborations in Mathematics and Geosciences program has helped. We can hope that SIAM will be a breeding ground for the "mathematics plus" that is so badly needed in geophysical fluid dynamics.

Acknowledgments

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[2] G.H. Roe and M.B. Baker, *Why is climate sensitivity so unpredictable?*, Science, 318 (2007), 629–632.

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