Math 355 Homework Problems #4

MATRIX ANALYSIS AND APPLIED LINEAR ALGEBRA, by C. Meyer

1. Here you will *numerically* solve the normal equations to fit curves to data. The use of MATLAB will be necessary. There are tutorials in the shared Google Drive folder. The data is also located in that folder.

(a) Import the data from the file "`LeastSquaresDataLinear.mat`". You will see two variables, x and y. Use least-squares to fit the data with the linear function $y = a_0 + a_1 x$. What are a_0, a_1? Plot the data, along with the least-squares curve, on one plot.

(b) Import the data from the file "`LeastSquaresDataQuadratic.mat`". You will see two variables, x and y. Use least-squares to fit the data with the quadratic function $y = a_0 + a_1 x + a_2 x^2$. What are a_0, a_1, a_2? Plot the data, along with the least-squares curve, on one plot.

2. The first four Legendre polynomials are given by

$$f_1(x) = 1,\ f_2(x) = x,\ f_3(x) = 3x^2 - 1,\ f_4(x) = 5x^3 - 3x.$$

(a) Show that $\{f_1, \ldots, f_4\}$ is a basis for $\mathbb{F}_3[x]$.

(b) If possible, find the unique linear combination of these Legendre polynomials which fit the data (1,3), (2,4), (3,1), (4,-2). If it is not possible, explain why.

3. Let $V \cong W$ with isomorphism $\mathcal{L} : V \mapsto W$.

(a) Show that if the set $\{\boldsymbol{v}_1, \ldots, \boldsymbol{v}_k\} \subset V$ is linearly independent, then so is the set $\{\mathcal{L}(\boldsymbol{v}_1), \ldots, \mathcal{L}(\boldsymbol{v}_k)\} \subset W$.

(b) Show that if the set $\{\boldsymbol{v}_1, \ldots, \boldsymbol{v}_n\} \subset V$ is a basis, then so is the set $\{\mathcal{L}(\boldsymbol{v}_1), \ldots, \mathcal{L}(\boldsymbol{v}_n)\} \subset W$.

(c) Show that $\dim[V] = \dim[W]$.