Math 355 Homework Problems #3

MATRIX ANALYSIS AND APPLIED LINEAR ALGEBRA, by C. Meyer

1. Consider the consistent linear system $\boldsymbol{Ax} = \boldsymbol{b}$.

(a) Let $\boldsymbol{a} \in \text{Col}(\boldsymbol{A}^{\text{H}})$ be given. Show that $\boldsymbol{a}^{\text{H}}\boldsymbol{x}$ is constant for *all* solutions to the system. *Note*: The result is clearly true if the solution is unique. You need to show that it is true if $\text{Null}(\boldsymbol{A})$ is nontrivial.

(b) Suppose that $\boldsymbol{y} \in \text{Null}(\boldsymbol{A}^{\text{H}})$. Show that $\boldsymbol{y}^{\text{H}}\boldsymbol{b} = 0$. (this result is known as the Fredholm alternative)

2. For the matrix and vector,

$$\boldsymbol{A} = \begin{pmatrix} 1 & 2 & 5 \\ 0 & -1 & -1 \\ 3 & -5 & 4 \end{pmatrix}, \quad \boldsymbol{b} = \begin{pmatrix} 5 \\ -7 \\ 3 \end{pmatrix},$$

write $\boldsymbol{b} = \boldsymbol{b}_{\text{c}} + \boldsymbol{b}_{\text{n}}$, where $\boldsymbol{b}_{\text{c}} \in \text{Col}(\boldsymbol{A})$ and $\boldsymbol{b}_{\text{n}} \in \text{Null}(\boldsymbol{A}^{\text{H}})$.

3. For the matrix

$$\boldsymbol{A} = \begin{pmatrix} 2 & 7 & 3 & 4 \\ 1 & 1 & -1 & -3 \\ 4 & 6 & -2 & -8 \end{pmatrix},$$

find $\text{Col}(\boldsymbol{A})$, $\text{Col}(\boldsymbol{A}^{\text{H}})$, $\text{Null}(\boldsymbol{A})$, $\text{Null}(\boldsymbol{A}^{\text{H}})$.

4. Set

$$\boldsymbol{A} = \begin{pmatrix} 1 & 2 & -1 \\ -1 & 1 & -5 \\ 3 & 4 & 1 \end{pmatrix}, \quad \boldsymbol{B} = \begin{pmatrix} 4 & 0 & 5 \\ -1 & 3 & 1 \\ 10 & -2 & 11 \end{pmatrix}.$$

Is $\text{Col}(\boldsymbol{A}) = \text{Col}(\boldsymbol{B})$? Explain. If the answer is YES, what is a minimal spanning set for $\text{Col}(\boldsymbol{A})$?