Math 355 Homework Problems #10

MATRIX ANALYSIS AND APPLIED LINEAR ALGEBRA, by C. Meyer

1. Consider the matrix

$$A = \begin{pmatrix} 3 & 2 & 0 \\ 2 & 3 & 0 \\ 0 & 0 & 4 \end{pmatrix} + \epsilon \begin{pmatrix} 1 & 0 & 3 \\ 0 & -2 & -7 \\ 3 & -7 & -4 \end{pmatrix}.$$

Label the eigenvalues as $\lambda_j = \lambda_j^0 + \epsilon\lambda_j^1 + \mathcal{O}(\epsilon^2)$ for $j = 1, 2, 3$. Determine the constants λ_j^1.

2. Consider the matrix and subspace,

$$A = \begin{pmatrix} -1 & 0 & 5 \\ 0 & 2 & 0 \\ 5 & 0 & -1 \end{pmatrix}, \quad S = \mathrm{Span}\left\{\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}\right\}.$$

Let P_S denote the orthogonal projection matrix onto S.

(a) Find a function whose zeros correspond to the eigenvalues for the 2×2 matrix induced by $P_s A P_S$.

(b) Explicitly find the eigenvalues for the induced matrix.

3. Let $A_0 \in \mathcal{M}_n(\mathbb{F})$ be simple and Hermitian. Let $V \in \mathcal{M}_{n\times k}$ for $1 \le k < n$ be an orthogonal matrix. Consider the matrix

$$A = A_0 + \epsilon V V^{\mathrm{H}}, \quad |\epsilon| \ll 1.$$

(a) Verify that A is Hermitian.

(b) Label the real eigenvalues as $\lambda_j = \lambda_j^0 + \epsilon\lambda_j^1 + \mathcal{O}(\epsilon^2)$ for $j = 1, \ldots, n$. Determine λ_j^1.

(c) Show that $\lambda_j^1 \ge 0$ for each j.

(d) What condition is needed for $\lambda_j^1 = 0$?

(e) How does the set of eigenvalues for A relate to the set of eigenvalues for A_0?