Math 355 Homework Problems #1

MATRIX ANALYSIS AND APPLIED LINEAR ALGEBRA, by C. Meyer

1. Find the coefficients for the cubic function $y = a_0 + a_1 x + a_2 x^2 + a_3 x^3$ which allow it to pass through the four points $(0,1)$, $(1,1)$, $(2,7)$, and $(3,31)$.

2. Consider the matrix $\boldsymbol{A} \in \mathcal{M}_4(\mathbb{F})$,

$$\boldsymbol{A} = \begin{pmatrix} 1 & 2 & -1 & -5 \\ 3 & 6 & 2 & 0 \\ -2 & -4 & 3 & 13 \\ 4 & 8 & 1 & -5 \end{pmatrix}.$$

(a) What is $\operatorname{rank}(\boldsymbol{A})$?

(b) Find all solutions to $\boldsymbol{Ax} = \boldsymbol{0}$.

3. Consider the boundary value problem,

$$y' = f(x), \quad y(0) = y(1) = 0.$$

The goal is to recast this BVP as a linear algebra problem. Pick $N \geq 1$, and discretize the unit interval via

$$x_j = jh,\ j = 0, 1, \ldots, N; \quad h = \frac{1}{N}.$$

Set

$$y_j = y(x_j), \quad f_j = f(x_j).$$

(a) Using the rule,

$$y'(x) \sim \frac{y(x+h) - y(x-h)}{2h},$$

find the matrix $\boldsymbol{D}$, and vectors $\boldsymbol{y}, \boldsymbol{f}$, so that the BVP is equivalent to $\boldsymbol{Dy} = \boldsymbol{f}$.

(b) Using the rule,

$$y'(x) \sim \frac{y(x+h) - y(x)}{h},$$

find the matrix $\boldsymbol{D}$, and vectors $\boldsymbol{y}, \boldsymbol{f}$, so that the BVP is equivalent to $\boldsymbol{Dy} = \boldsymbol{f}$.

4. Let $\boldsymbol{A} \in \mathcal{M}_n(\mathbb{F})$ be a square matrix. Show that:

(a) $\boldsymbol{A} + \boldsymbol{A}^{\mathrm{H}}$ is a Hermitian matrix

(b) $\boldsymbol{A} - \boldsymbol{A}^{\mathrm{H}}$ is a skew-Hermitian matrix.

5. Show that any matrix $\boldsymbol{A} \in \mathcal{M}_n(\mathbb{F})$ can be written as the sum of a Hermitian matrix and a skew-Hermitian matrix.

6. Let $\boldsymbol{A} \in \mathcal{M}_{m\times n}(\mathbb{F})$. Show that:

(a) $\boldsymbol{A}\boldsymbol{A}^{\mathrm{H}}$ is Hermitian

(b) $\boldsymbol{A}^{\mathrm{H}}\boldsymbol{A}$ is Hermitian.