

Math 355 Homework Problems #0

Notation. In all that follows, $\mathbb{F} = \mathbb{R}$, or \mathbb{C} . The notation is used when it makes no difference if the numbers are real-valued or complex-valued. The symbol $\mathcal{M}_{m \times n}(\mathbb{F})$ represents the set of all matrices with m rows and n columns where the entries are in \mathbb{F} . Finally, the symbol \mathbb{F}^n represents the set of all vectors of length n with entries in \mathbb{F} .

1. Consider the homogeneous system $A\mathbf{x} = \mathbf{0}$, where $A \in \mathcal{M}_{m \times n}(\mathbb{F})$ with $m < n$. Explain why this system must always have an infinite number of solutions.

2. Consider the nonhomogeneous system $A\mathbf{x} = \mathbf{b}$, where $A \in \mathcal{M}_{m \times n}(\mathbb{F})$ and $\mathbf{b} \in \mathbb{F}^m$ is nonzero.

(a) If \mathbf{x}_1 and \mathbf{x}_2 are two solutions, must it be the case that $3\mathbf{x}_1 - 4\mathbf{x}_2$ is also a solution? Why, or why not?

(b) Suppose that $m \geq n$, and further suppose that the system is consistent. What must the row-reduced matrix look like if the solution is unique?

3. Find all of the solutions to the system

$$\begin{aligned}x - 3y - 4z &= -6 \\2x + 4z &= -6 \\-6x + 4y + 4z &= 22.\end{aligned}$$

If the system is not consistent, state why.

4. Find eigenvalues and associated eigenvectors for the following matrices:

(a) $\begin{pmatrix} 0 & 1 \\ -2 & -3 \end{pmatrix}$

(b) $\begin{pmatrix} 0 & 1 & 0 \\ -6 & 5 & 0 \\ 3 & 4 & 7 \end{pmatrix}$