**Math 355 Homework Problems #1**  
*Matrix Analysis and Applied Linear Algebra, by C. Meyer*

1. Consider a cubic function of the form $y = a_0 + a_1 x + a_2 x^2 + a_3 x^3$.

   (a) Is it possible for this function to pass through the three points $(0, 1), (1, 1)$, and $(2, 7)$? If so, is the function unique? If not, why not?

   (b) Is it possible for this function to pass through the four points $(0, 1), (1, 1), (2, 7)$, and $(3, 31)$? If so, is the function unique? If not, why not?

   (c) Is it possible for this function to pass through the five points $(-2, -29), (-1, -5), (0, 1), (1, 1)$, and $(2, 7)$? If so, is the function unique? If not, why not?

2. Consider the homogeneous system $Ax = 0$, where $A \in \mathbb{R}^{m \times n}$ with $m < n$. Explain why this system must always have an infinite number of solutions.

3. Consider the nonhomogeneous system $Ax = b$, where $A \in \mathbb{R}^{m \times n}$ and $b \in \mathbb{R}^m$ is nonzero.

   (a) If $x_1$ and $x_2$ are two solutions, must it be the case that $3x_1 - 4x_2$ is also a solution? Why, or why not?

   (b) Suppose that $m \geq n$, and further suppose that the system is consistent. What must the row-reduced matrix $E_A$ look like if the solution is unique?

   (c) Suppose that $m < n$, and further suppose that the system is consistent. Is it possible for the solution to be unique? Why, or why not?

4. Find all of the solutions to the system

   $\begin{align*}
   x - 3y - 4z &= -6 \\
   2x + 4z &= -6 \\
   -6x + 4y + 4z &= 22.
   \end{align*}$

   If the system is not consistent, state why.

5. Let $A \in \mathbb{R}^{n \times n}$ be a square matrix. Show that:

   (a) $A + A^T$ is a symmetric matrix

   (b) $A - A^T$ is a skew-symmetric matrix.

6. Show that any matrix $A \in \mathbb{R}^{n \times n}$ can be written as the sum of a symmetric matrix and a skew-symmetric matrix.

7. For $x \in \mathbb{R}^n$, show that $x^T x = 0$ if and only if $x = 0$. 

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