Math 355 Homework Problems #6

1. Find an orthonormal basis for each of the four fundamental subspaces of

$$\boldsymbol{A} = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 4 & 2 \\ 0 & 5 & 5 \\ 3 & -4 & 2 \end{pmatrix} \in \mathcal{M}_{4\times 3}(\mathbb{R}).$$

2. Find the quadratic curve of best fit for the data $(-1,2),(0,-1),(1,1),(2,3),(3,7),(1,-2),(2,5)$.

3. Let the inner product on $\mathcal{M}_n(\mathbb{F})$ be given by

$$\langle \boldsymbol{A}, \boldsymbol{B} \rangle = \mathrm{trace}(\boldsymbol{A}^{\mathrm{H}}\boldsymbol{B}) = \sum_{j=1}^{n} \boldsymbol{a}_j^{\mathrm{H}} \boldsymbol{b}_j,$$

where $\boldsymbol{A} = (\boldsymbol{a}_1\ \boldsymbol{a}_2\ \cdots\ \boldsymbol{a}_n)$ and $\boldsymbol{B} = (\boldsymbol{b}_1\ \boldsymbol{b}_2\ \cdots\ \boldsymbol{b}_n)$. Let $\boldsymbol{E}_{jk} \in \mathcal{M}_n(\mathbb{F})$ for $j,k = 1,\ldots,n$ be the matrix with a one in the jk entry, and zero everywhere else. Show that:

(a) each $\boldsymbol{E}_{jk}$ is a rank one matrix

(b) $\langle \boldsymbol{E}_{jk}, \boldsymbol{E}_{\ell m} \rangle = \begin{cases} 0, & (\ell,m) \neq (j,k) \\ 1, & (\ell,m) = (j,k) \end{cases}$

(c) $\{\boldsymbol{E}_{jk}\}$ is an orthonormal basis for $\mathcal{M}_n(\mathbb{F})$.

4. Suppose that $\boldsymbol{A},\boldsymbol{B} \in \mathcal{M}_n(\mathbb{F})$ are similar, i.e., $\boldsymbol{A} = \boldsymbol{P}^{-1}\boldsymbol{B}\boldsymbol{P}$ for some invertible matrix $\boldsymbol{P}$. Show that for any integer k, $\boldsymbol{A}^k = \boldsymbol{P}^{-1}\boldsymbol{B}^k\boldsymbol{P}$.