Math 355 Homework Problems #5

1. Let $S \subset \mathbb{F}^n$ be a subspace with orthonormal basis $\{\boldsymbol{u}_1, \boldsymbol{u}_2, \ldots, \boldsymbol{u}_k\}$. Let

$$\boldsymbol{P}_S = \boldsymbol{U}\boldsymbol{U}^{\mathrm{H}}, \quad \boldsymbol{U} = (\boldsymbol{u}_1\ \boldsymbol{u}_2\ \cdots\ \boldsymbol{u}_k),$$

be the orthogonal projection matrix using this basis. Set

$$\boldsymbol{P}_{S^\perp} = \boldsymbol{I}_n - \boldsymbol{P}_S.$$

Show that:

(a) $\boldsymbol{P}_{S^\perp} \cdot \boldsymbol{P}_{S^\perp} = \boldsymbol{P}_{S^\perp}$

(b) $\ker(\boldsymbol{P}_{S^\perp}) = S$

(c) $\mathrm{Ran}(\boldsymbol{P}_{S^\perp}) = S^\perp$

(d) $\boldsymbol{P}_S \cdot \boldsymbol{P}_{S^\perp} = \boldsymbol{P}_{S^\perp} \cdot \boldsymbol{P}_S = \boldsymbol{0}_n$.

2. Let $\{1 + 3x^2, 1 - 6x^2\}$ be a basis for a subspace S of the vector space $\mathbb{R}_2[x]$, which has the inner product

$$\langle f, g\rangle = \int_0^1 f(x)g(x)\,\mathrm{d}x.$$

Find the orthogonal projection of x onto S.

3. Find the QR-factorization for

$$\boldsymbol{A} = \begin{pmatrix} 1 & -1 \\ 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{pmatrix}.$$