Math 355 Homework Problems #1

1. Do the following form subspaces of $\mathbb{F}_4[x]$? Prove, or give a counter-example.

(a) The subset of all polynomials in $\mathbb{F}_4[x]$ of even degree.

(b) The subset of all polynomials $p(x)$ in $\mathbb{F}_4[x]$ with $p(0) = 0$.

(c) The subset of all polynomials $p(x)$ in $\mathbb{F}_4[x]$ with $p'(0) \neq 0$.

2. Which of each set below is a spanning set for $\mathbb{F}_2[x]$? Justify your answer in each case.

(a) $\{1, x-1, x^2+1\}$.

(b) $\{x+2, x-2, x^2-2\}$.

(c) $\{1+2x+3x^2, 4+5x+6x^2, 7+8x+9x^2\}$.

3. If $\boldsymbol{A} \in \mathcal{M}_n(\mathbb{F})$, the transpose of $\boldsymbol{A}$, denoted $\boldsymbol{A}^{\mathrm{T}}$, is the matrix where the j^{th} column of $\boldsymbol{A}$ is the j^{th} row of $\boldsymbol{A}^{\mathrm{T}}$. Let $\mathrm{Sym}_n(\mathbb{F}) \subset \mathcal{M}_n(\mathbb{F})$ denote the set of symmetric matrices. In other words, $\boldsymbol{A} \in \mathrm{Sym}_n(\mathbb{F})$ if and only if $\boldsymbol{A} = \boldsymbol{A}^{\mathrm{T}}$. Let $\mathrm{Skew}_n(\mathbb{F}) \subset \mathcal{M}_n(\mathbb{F})$ denote the set of skew-symmetric matrices. In other words, $\boldsymbol{A} \in \mathrm{Skew}_n(\mathbb{F})$ if and only if $\boldsymbol{A} = -\boldsymbol{A}^{\mathrm{T}}$.

(a) Show that $\mathrm{Sym}_n(\mathbb{F})$ is a subspace.

(b) Show that $\mathrm{Skew}_n(\mathbb{F})$ is a subspace.

(c) Show that $\mathcal{M}_n(\mathbb{F}) = \mathrm{Sym}_n(\mathbb{F}) \oplus \mathrm{Skew}_n(\mathbb{F})$.

4. Write $\mathbb{F}_5[x]$ as the direct sum of 6 one-dimensional subspaces.

5. Write $\mathcal{M}_2(\mathbb{F})$ as the direct sum of 4 one-dimensional subspaces.

6. Let $W_1, W_2, \ldots, W_n$ be a collection of subspaces of the vector space V. Show that

$$\bigcap_{j=1}^{n} W_j = W_1 \cap W_2 \cap \cdots \cap W_n$$

is a subspace.