Math 355 Homework Problems #0

1. Consider a cubic function of the form $y = a_0 + a_1 x + a_2 x^2 + a_3 x^3$.

(a) Is it possible for this function to pass through the three points $(0,1)$, $(1,1)$, and $(2,7)$? If so, is the function unique? If not, why not?

(b) Is it possible for this function to pass through the four points $(0,1)$, $(1,1)$, $(2,7)$, and $(3,31)$? If so, is the function unique? If not, why not?

(c) Is it possible for this function to pass through the five points $(-2,-29)$, $(-1,-5)$, $(0,1)$, $(1,1)$, and $(2,7)$? If so, is the function unique? If not, why not?

2. Consider the homogeneous system $\boldsymbol{Ax} = \boldsymbol{0}$, where $\boldsymbol{A} \in \mathcal{M}_{m\times n}$ with $m < n$. Explain why this system must always have an infinite number of solutions.

3. Consider the nonhomogeneous system $\boldsymbol{Ax} = \boldsymbol{b}$, where $\boldsymbol{A} \in \mathcal{M}_{m\times n}(\mathbb{F})$ and $\boldsymbol{b} \in \mathbb{F}^m$ is nonzero.

(a) If $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ are two solutions, must it be the case that $3\boldsymbol{x}_1 - 4\boldsymbol{x}_2$ is also a solution? Why, or why not?

(b) Suppose that $m \ge n$, and further suppose that the system is consistent. What must the row-reduced matrix look like if the solution is unique?

4. Find all of the solutions to the system

$$\begin{aligned} x - 3y - 4z &= -6 \\ 2x + 4z &= -6 \\ -6x + 4y + 4z &= 22. \end{aligned}$$

If the system is not consistent, state why.

5. Find eigenvalues and associated eigenvectors for the following matrices:

(a) $\begin{pmatrix} 0 & 1 \\ -2 & -3 \end{pmatrix}$

(b) $\begin{pmatrix} 0 & 1 & 0 \\ -6 & 5 & 0 \\ 3 & 4 & 7 \end{pmatrix}$