Math 333 Homework Problems #1

Applied Partial Differential Equations: A Dynamics Perspective

1.8.2, **1.8.6**, **1.8.4**, **1.8.7**, **1.8.8**

4. This is a revision of Problem **1.8.1**. Consider the random walk derivation given in Chapter 1.2.2.

(a) Suppose the probability of moving left is p_ℓ, and the probability of moving right is $p_r = 1 - p_\ell$. Assume $p_r \neq p_\ell$. If $\Delta t = A\Delta x$, show that in the limit $\Delta x \to 0$ the governing PDE is an advection equation. What is the speed of propagation, c?

(b) In part (a) suppose that $p_\ell - p_r = B\Delta x$, where $B \neq 0$. If $\Delta t = (\Delta x)^2/(2D)$, show that in the limit $\Delta x \to 0$ the governing PDE is an advection-diffusion equation. What is the speed of propagation, c?

(c) Let p_ℓ be the probability of moving left, p_r the probability of moving right, and p_s the probability of staying. The probabilities sum to one. Assume $p_s = A(\Delta x)^2$, $p_\ell - p_r = B\Delta x$, where $A, B \neq 0$. If $\Delta t = (\Delta x)^2/(2D)$, show that in the limit $\Delta x \to 0$ the governing PDE is an advection-diffusion-growth PDE.