Math 333 Homework Problems #4

APPLIED PARTIAL DIFFERENTIAL EQUATIONS (3RD EDITION), by J.D. Logan

1. Consider the SL problem,

$$[p(x)v']' - q(x)v = \lambda w(x)v,$$

with the separated boundary conditions,

$$a_0 v(0) + a_1 v'(0) = 0, \quad b_0 v(1) + b_1 v'(1) = 0.$$

Assume $a_0, b_0, b_1 \geq 0, a_1 \leq 0$, and $q(x) \geq 0$. Show that:

(a) the eigenvalues are nonpositive

(b) if $a_0 > 0$ and/or $b_0 > 0$, then the eigenvalues are strictly negative.

2. Find the eigenvalues and associated normalized eigenfunctions for the SL problem,

$$v'' = \lambda v, \quad v(0) = v'(1) = 0.$$

3. Find the eigenvalues and associated normalized eigenfunctions for the SL problem,

$$v'' = \lambda v, \quad v'(0) = v(1) = 0.$$

4. Consider the two SL problems,

$$[p(x)v']\, v' = \lambda v,$$

and

$$[p(x)v']\, v' - q_0^2 v = \lambda v,$$

where $q_0 > 0$ is a constant. Suppose that each problem has the same separated boundary conditions,

$$a_0 v(0) + a_1 v'(0) = 0, \quad b_0 v(1) + b_1 v'(1) = 0$$

Show that if λ_0 is an eigenvalue for the first problem with associated eigenfunction v_0, then $\lambda_0 - q_0^2$ is an eigenvalue for the second problem with the same associated eigenfunction v_0.

5. Here we explore the effect of the diffusion coefficient on the value of the eigenvalues. Consider the two SL problems,

$$kv'' = \lambda v,$$

and

$$v'' = \lambda v.$$

Suppose that each problem has the same separated boundary conditions,

$$a_0 v(0) + a_1 v'(0) = 0, \quad b_0 v(1) + b_1 v'(1) = 0$$

How do the eigenvalues/eigenfunctions of the first problem relate to those of the second? (*Hint*: rescale the spectral parameter in the first eigenvalue problem)