Math 333 Homework Problems #8

APPLIED PARTIAL DIFFERENTIAL EQUATIONS (2ND EDITION), by J.D. Logan

4.5. Diffusion in a disk

• 4.5.6 Consider the heat equation on the quarter-circle of radius $R$:

$$ u_t = k \Delta u $$

$$ u_\theta(r, 0, t) = u_\theta(r, \pi/2, t) = u(R, \theta, t) = 0 $$

$$ u(r, \theta, 0) = f(r, \theta). $$

Find the solution, and analyze the temperature as $t \to \infty$.

• 4.5.7 Consider the wave equation on the half-circle of radius $R$:

$$ u_{tt} = c^2 \Delta u $$

$$ u(r, 0, t) = u_\theta(r, \pi, t) = u(R, \theta, t) = 0 $$

$$ u(r, \theta, 0) = f(r, \theta), \; u_t(r, \theta, 0) = 0. $$

Solve the initial value problem.

• 4.5.8 Consider the forced heat equation on the wedge of radius $R$:

$$ u_t = k \Delta u $$

$$ u_\theta(r, 0, t) = u(r, \pi/3, t) = 0, \; u(R, \theta, t) = h(\theta) $$

$$ u(r, \theta, 0) = f(r, \theta). $$

Find the solution, and analyze the temperature as $t \to \infty$. 