

Math 333 Homework Problems #8

APPLIED PARTIAL DIFFERENTIAL EQUATIONS (2ND EDITION), by J.D. Logan

4.5. Diffusion in a disk

- **4.5.6** Consider the heat equation on the quarter-circle of radius R :

$$\begin{aligned}u_t &= k\Delta u \\u_\theta(r, 0, t) &= u_\theta(r, \pi/2, t) = u(R, \theta, t) = 0 \\u(r, \theta, 0) &= f(r, \theta).\end{aligned}$$

Find the solution, and analyze the temperature as $t \rightarrow \infty$.

- **4.5.7** Consider the wave equation on the half-circle of radius R :

$$\begin{aligned}u_{tt} &= c^2\Delta u \\u(r, 0, t) &= u_\theta(r, \pi, t) = u(R, \theta, t) = 0 \\u(r, \theta, 0) &= f(r, \theta), \quad u_t(r, \theta, 0) = 0.\end{aligned}$$

Solve the initial value problem.

- **4.5.8** Consider the forced heat equation on the wedge of radius R :

$$\begin{aligned}u_t &= k\Delta u \\u_\theta(r, 0, t) &= u(r, \pi/3, t) = 0, \quad u(R, \theta, t) = h(\theta) \\u(r, \theta, 0) &= f(r, \theta).\end{aligned}$$

Find the solution, and analyze the temperature as $t \rightarrow \infty$.