Math 333 Homework Problems #7

APPLIED PARTIAL DIFFERENTIAL EQUATIONS (2ND EDITION), by J.D. Logan

4.2. Flux and radiation conditions

- 4.2.5 Consider the heat equation on the rectangle $0 < x < L$, $0 < y < H$:

$$\begin{aligned} u_t &= k\Delta u \\ u_x(0,y,t) &= u_x(L,y,t) = u_y(x,0,t) = u_y(x,H,t) = 0 \\ u(x,y,0) &= f(x,y). \end{aligned}$$

  Find the solution, and analyze the temperature as $t \to \infty$.

- 4.2.6 Consider the wave equation on the rectangle $0 < x < L$, $0 < y < H$:

$$\begin{aligned} u_{tt} &= c^2\Delta u \\ u(0,y,t) &= u(L,y,t) = u_y(x,0,t) = u_y(x,H,t) = 0 \\ u(x,y,0) &= 0, \;\; u_t(x,y,0) = f(x,y). \end{aligned}$$

  Solve the initial value problem.

- 4.2.7 Consider Poisson's equation on the rectangle $0 < x < L$, $0 < y < H$:

$$\begin{aligned} -\Delta u &= f(x,y) \\ u_x(0,y) &= u_x(L,y) = u(x,0) = u(x,H) = 0. \end{aligned}$$

  Find the series solution. Plot the solution for the special case of

$$L = 2\pi,\; H = \pi \quad f(x,y) = \mathrm{e}^{-3(x-L/2)^2-3(y-H/2)^2}.$$

- 4.2.8 Consider the forced heat equation on the rectangle $0 < x < L$, $0 < y < H$:

$$\begin{aligned} u_t &= \Delta u + q(x,y) \\ u_x(0,y,t) &= u_x(L,y,t) = u(x,0,t) = 0,\; u(x,H,t) = f_{\mathrm{T}}(x). \end{aligned}$$

  Find the solution, and analyze the temperature as $t \to \infty$.