Math 333 Homework Problems #4

APPLIED PARTIAL DIFFERENTIAL EQUATIONS (2ND EDITION), by J.D. Logan

4.1. Separation of variables

- 4.1.4

4.2. Flux and radiation conditions

- 4.2.1, 4.2.4
- 4.2.5 Consider the heat equation

$$\begin{aligned} u_t &= u_{xx} + \cos(\pi t)\sin(3x), \quad 0 < x < \pi/2 \\ u(0,t) &= u_x(\pi/2,t) = 0 \\ u(x,0) &= \frac{x(\pi - x)}{(\pi/2)^2}. \end{aligned}$$

  Find the solution $u(x,t)$, and determine $\lim_{t\to+\infty} u(x,t)$. Plot the solution for $0 \le t \le 15$.

- 4.2.6 Consider the wave equation

$$\begin{aligned} u_{tt} &= u_{xx} + \sin(\omega t)\cos(2x), \quad 0 < x < \pi \\ u_x(0,t) &= u_x(\pi,t) = 0 \\ u(x,0) &= \frac{x^4(\pi - x)^4}{(\pi/2)^8}, \quad u_t(x,0) = 0. \end{aligned}$$

  (a) Suppose that $\omega = 1$. Find the solution $u(x,t)$, and determine the dominant behavior (if any) for $\lim_{t\to+\infty} u(x,t)$. Plot the solution for $0 \le t \le 30$.

  (b) Suppose that $\omega = 1.9$. Find the solution $u(x,t)$, and determine the dominant behavior (if any) for $\lim_{t\to+\infty} u(x,t)$. Plot the solution for $0 \le t \le 30$.

  (c) Suppose that $\omega = 2$. Find the solution $u(x,t)$, and determine the dominant behavior (if any) for $\lim_{t\to+\infty} u(x,t)$. Plot the solution for $0 \le t \le 30$.