Math 333 Homework Problems #3

APPLIED PARTIAL DIFFERENTIAL EQUATIONS (2ND EDITION), by J.D. Logan

3.4. Sturm-Liouville problems

- 3.4.3, 3.4.4 ,3.4.7, 3.4.8
- 3.4.14 Consider the SLP with mixed Dirichlet and Neumann boundary conditions:

$$-y'' = \lambda y,\ 0 < x < L; \quad y(0) = 0,\ y'(L) = 0.$$

  (a) Find the eigenvalues and corresponding eigenfunctions.

  (b) If the eigenfunctions are denoted by $y_n(x)$ for $n = 1, 2, 3, \ldots$, find the coefficients c_n so that

$$x(x - 2L) = \sum_{n=1}^{\infty} c_n y_n(x)$$

  in the mean-square sense.

- 3.4.15 Consider the SLP

$$-y'' = \lambda y,\ 0 < x < 2\pi; \quad \alpha y(0) + y'(0) = 0,\ \beta y(2\pi) + y'(2\pi) = 0.$$

  (a) Suppose that $\alpha = \beta$. Show that there is precisely one negative eigenvalue.

  (b) Show that if $\alpha = 0$, then there is precisely one negative eigenvalue if $\beta < 0$, and no negative eigenvalues if $\beta > 0$.

  (c) Show that if $\beta = 0$, then there is precisely one negative eigenvalue if $\alpha > 0$, and no negative eigenvalues if $\alpha < 0$.

  (d) What condition on α and β ensures that $\lambda = 0$ is an eigenvalue?

  (e) How many positive eigenvalues are there?