Math 333 Homework Problems #3
APPLIED PARTIAL DIFFERENTIAL EQUATIONS (2ND EDITION), by J.D. Logan

3.4. Sturm-Liouville problems

• 3.4.3, 3.4.4, 3.4.7, 3.4.8

• 3.4.14 Consider the SLP with mixed Dirichlet and Neumann boundary conditions:
\[ -y'' = \lambda y, \quad 0 < x < L; \quad y(0) = 0, \quad y'(L) = 0. \]

(a) Find the eigenvalues and corresponding eigenfunctions.
(b) If the eigenfunctions are denoted by \( y_n(x) \) for \( n = 1, 2, 3, \ldots \), find the coefficients \( c_n \) so that
\[ x(x - 2L) = \sum_{n=1}^{\infty} c_n y_n(x) \]
in the mean-square sense.

• 3.4.15 Consider the SLP
\[ -y'' = \lambda y, \quad 0 < x < 2\pi; \quad \alpha y(0) + y'(0) = 0, \quad \beta y(2\pi) + y'(2\pi) = 0. \]

(a) Suppose that \( \alpha = \beta \). Show that there is precisely one negative eigenvalue.
(b) Show that if \( \alpha = 0 \), then there is precisely one negative eigenvalue if \( \beta < 0 \), and no negative eigenvalues if \( \beta > 0 \).
(c) Show that if \( \beta = 0 \), then there is precisely one negative eigenvalue if \( \alpha > 0 \), and no negative eigenvalues if \( \alpha < 0 \).
(d) What condition on \( \alpha \) and \( \beta \) ensures that \( \lambda = 0 \) is an eigenvalue?
(e) How many positive eigenvalues are there?