

Math 333 Homework Problems #1

APPLIED PARTIAL DIFFERENTIAL EQUATIONS (2ND EDITION), by J.D. Logan

Those problems marked in red are to be turned in to be graded.

1. THE PHYSICAL ORIGINS OF PARTIAL DIFFERENTIAL EQUATIONS

- 1.3.5, 1.3.6
- 1.5.1, 1.5.2, 1.5.3, 1.5.4
- 1.5.9 This is an expanded version of **1.5.5**. Consider the wave equation

$$u_{tt} = c^2 u_{xx}, \quad 0 \leq x \leq L.$$

Define the total energy by

$$E(t) = \frac{1}{2} \int_0^L (u_t^2 + c^2 u_x^2) dx.$$

(a) Show that

$$E'(t) = c^2 u_x u_t \Big|_{x=0}^{x=L}.$$

(b) If $u(0, t) = u(L, t) = 0$, show that $E'(t) \equiv 0$.

(c) If $u_x(0, t) = u_x(L, t) = 0$, show that $E'(t) \equiv 0$.