Math 333 Homework Problems #6
APPLIED PARTIAL DIFFERENTIAL EQUATIONS ... (4TH EDITION), by R. Haberman

3. FOURIER SERIES

3.4. Term-by-term differentiation of Fourier series

- [3.4.12, 3.4.13]
- [3.4.14] Consider the heat equation

\[
\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + q(x,t); \quad \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(L,t) = 0, \quad u(x,0) = f(x).
\]

Assuming that \( q(x,t) \) is a piecewise smooth function of \( x \) for each \( t \geq 0 \), solve the problem. If

\[
q(x,t) := Q(x), \quad \int_0^L Q(x) \, dx = 0,
\]

determine \( \lim_{t \to +\infty} u(x,t) \).

3.5. Term-by-term integration of Fourier series

- [3.5.1, 3.5.2]