Solve a first-order scalar ODE

```
syms x(t)
eqn1 = t*diff(x(t),t) == x(t) + 3*t^2*cos(2 * t)
```

eqn1 =

$$t \frac{\partial}{\partial t} x(t) = x(t) + 3\, t^2 \cos(2\, t)$$

```
%sol1 = dsolve(eqn1)
ic1 = x(pi) == -4
```

ic1 = $x(\pi) = -4$

```
sol1 = simplify(dsolve(eqn1,ic1))
```

sol1 =

$$\frac{3\, t \sin(2\, t)}{2} - \frac{4\, t}{\pi}$$

Solve a second-order ODE

```
syms y(t)
Dy(t) = diff(y(t),t); D2y(t) = diff(y(t),t,2);
eqn2 = D2y(t) + 4*Dy(t) + 13*y(t) == 18*exp(-2*t) + 136*cos(5*t)
```

eqn2 =

$$\frac{\partial^2}{\partial t^2} y(t) + 4 \frac{\partial}{\partial t} y(t) + 13\, y(t) = 136 \cos(5\, t) + 18\, e^{-2\, t}$$

```
sol2 = simplify(dsolve(eqn2))
```

sol2 = $2\, e^{-2\, t} - 3 \cos(5\, t) + 5 \sin(5\, t) + C_{11} \cos(3\, t)\, e^{-2\, t} + C_{12} \sin(3\, t)\, e^{-2\, t}$

```
%ic2 = [y(0) == 4, Dy(0) == -1];
%sol2(t) = simplify(dsolve(eqn2,ic2))
%fplot(sol2(t))
```

Solve a system of ODEs

```
syms x(t) y(t)
eqn3 = [diff(x(t),t) == y(t) + exp(-t), diff(y(t),t) == -2*x(t) - 3*y(t) + 4]
```

eqn3 =

$$\left(\frac{\partial}{\partial t} x(t) = e^{-t} + y(t) \quad \frac{\partial}{\partial t} y(t) = 4 - 3\, y(t) - 2\, x(t) \right)$$

```
[sol3x,sol3y] = dsolve(eqn3);
```

```
x3 = simplify(sol3x)
```

x3 =

$$2\,t\,e^{-t} - e^{-t} - C_{13}\,e^{-t} - \frac{C_{14}\,e^{-2\,t}}{2} + 2$$

```
y3 = simplify(sol3y)
```

y3 = $e^{-2\,t}\,\left(C_{14} + 2\,e^{t} + C_{13}\,e^{t} - 2\,t\,e^{t}\right)$

```
%ic3 = [x(0) == 2, y(0) == -5]
%[sol3x,sol3y] = dsolve(eqn3,ic3);
%x3(t) = simplify(sol3x)
%y3(t) = simplify(sol3y)
%fplot(x3(t),'b-')
%hold on
%fplot(y3(t),'ro')
%hold off
%xlim([0 5])
```

Do partial fractions

```
syms s
f = (3*s-1)/(s^2+6*s+8)
```

f =

$$\frac{3\,s - 1}{s^2 + 6\,s + 8}$$

```
fp = partfrac(f)
```

fp =

$$\frac{13}{2\,\left(s + 4\right)} - \frac{7}{2\,\left(s + 2\right)}$$

Solve ODEs with Heaviside and Dirac delta functions

```
syms y(t)
sympref('HeavisideAtOrigin',1);
Dy(t) = diff(y(t),t); D2y(t) = diff(y(t),t,2);
eqn4 = D2y(t) + 7*Dy(t) + 10*y(t) == 3*heaviside(t-1) - 5*dirac(t-2)
```

eqn4 =

$$\frac{\partial^2}{\partial t^2}\,y(t) + 7\,\frac{\partial}{\partial t}\,y(t) + 10\,y(t) = 3\,\mathrm{heaviside}(t - 1) - 5\,\delta(t - 2)$$

```
%sol4 = simplify(dsolve(eqn4,'IgnoreAnalyticConstraints',false))
ic4 = [y(0) == 0, Dy(0) == 0];
sol4(t) = simplify(dsolve(eqn4,ic4,'IgnoreAnalyticConstraints',false))
```

```
sol4(t) =
```

$$e^{-5t}\left(\frac{5\,\mathrm{heaviside}(t-2)\,e^{10}}{3}-\frac{\sigma_1\,(e^{5t}-e^{5})}{5}\right)-e^{-2t}\left(\frac{5\,\mathrm{heaviside}(t-2)\,e^{4}}{3}-\frac{\sigma_1\,(e^{2t}-e^{2})}{2}\right)$$

where

$$\sigma_1=\frac{\mathrm{sign}(t-1)}{2}+\frac{1}{2}$$

```
fplot(sol4(t),[0 6])
```