Math 231 Homework Problems #8

DIFFERENTIAL EQUATIONS WITH LINEAR ALGEBRA, by M. Boelkins, M. Potter, and J. Goldberg

3.8. Applications of linear systems

13 Consider the system of two interconnected tanks given below:



The tank on the left initially has 50 g of salt present in its solution, while the tank on the right has 40 g in its solution.

- (a) Set up the initial value problem whose solution will determine the amount of salt in each tank.
- (b) After a very long time does the amount of salt in each tank substantively depend on the initial amount of salt in each tank?
- (c) Suppose that

$$c_1(t) = 30, \quad c_2(t) = 15.$$

Find the limiting concentration in each tank.

(d) Suppose that

$$c_1(t) = 30 + 10\sin t$$
, $c_2(t) = 15 + 10\cos t$.

Find the limiting concentration in each tank. Using the identity

$$\cos(\theta_1 - \theta_2) = \cos\theta_1 \cos\theta_2 + \sin\theta_1 \sin\theta_2,$$

determine the maximum and minimum concentrations in each tank. Discuss how the maximum and minimum concentrations in each tank relate to each other.