Math 231 Homework Problems #8
DIFFERENTIAL EQUATIONS WITH LINEAR ALGEBRA, by M. Boelkins, M. Potter, and J. Goldberg

3.8. Applications of linear systems

Consider the system of two interconnected tanks given below:

\[ \begin{align*}
10 \text{ L/m} & \quad c_1(t) \text{ g/L} \\
5 \text{ L/m} & \quad 100 \text{ L} \\
5 \text{ L/m} & \quad 10 \text{ L/m} \\
100 \text{ L} & \quad 100 \text{ L} \\
5 \text{ L/m} & \quad 10 \text{ L/m} \\
5 \text{ L/m} & \quad c_2(t) \text{ g/L} \\
10 \text{ L/m} & \quad 100 \text{ L}
\end{align*} \]

The tank on the left initially has 50 g of salt present in its solution, while the tank on the right has 40 g in its solution.

(a) Carefully read Section 3.4.3 in the online lecture notes.
(b) Set up the initial value problem whose solution will determine the amount of salt in each tank.
(c) After a very long time does the amount of salt in each tank substantively depend on the initial amount of salt in each tank?
(d) Suppose that \( c_1(t) = 30, \quad c_2(t) = 15. \)
    Find the limiting concentration in each tank.
(e) Suppose that \( c_1(t) = 30 + 10 \sin t, \quad c_2(t) = 15 + 10 \cos t. \)
    Find the limiting concentration in each tank. Using the identity
    \[ \cos(\theta_1 - \theta_2) = \cos \theta_1 \cos \theta_2 + \sin \theta_1 \sin \theta_2, \]
    determine the maximum and minimum concentrations in each tank. Discuss how the maximum and minimum concentrations in each tank relate to each other.

Suppose that the initial condition for the coupled oscillator problem discussed in Section 3.4.2 in the online lecture notes is now given by
\[ \begin{align*}
y_1(0) = 0, \quad y_1'(0) = F; \\
y_2(0) = y_2'(0) = 0.
\end{align*} \]

(a) Solve the IVP.
(b) Is there still an alternating of the beating phenomena? Explain.