

1. For the following matrices find a basis for $\text{Col}(A)$ and $\text{Null}(A)$:

$$(a) \quad A = \begin{pmatrix} 2 & 3 \\ -4 & -6 \end{pmatrix} \quad (b) \quad A = \begin{pmatrix} 1 & -2 & 3 \\ -1 & 2 & -2 \\ 2 & -4 & -5 \end{pmatrix} \quad (c) \quad A = \begin{pmatrix} 1 & 2 & 2 & 3 \\ 2 & 5 & 4 & 8 \\ -1 & -3 & -2 & -5 \\ 0 & 2 & 0 & 4 \end{pmatrix}$$

2. Compute $\det(A)$, and find a basis for $\text{Null}(A)$, for each of the following matrices:

$$(a) \quad A = \begin{pmatrix} 2 & 3 \\ 4 & -6 \end{pmatrix} \quad (b) \quad A = \begin{pmatrix} 2 & -1 & 2 \\ 1 & 3 & 2 \\ 5 & 1 & 6 \end{pmatrix} \quad (c) \quad A = \begin{pmatrix} 2 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 6 & 2 & 0 \\ 1 & 1 & -2 & 3 \end{pmatrix}$$

3. For the following problems write the solution in the form $x = x_h + x_p$, where $x_h \in \text{Null}(A)$ and x_p is a particular solution. If there is no solution, state why.

$$(a) \quad \begin{array}{rcl} 2x_1 + 3x_2 + x_3 & = & 1 \\ x_1 + x_2 + x_3 & = & 3 \\ 3x_1 + 4x_2 + 2x_3 & = & 4 \end{array} \quad (b) \quad \begin{array}{rcl} x_1 + x_2 + 3x_3 & = & 2 \\ x_1 + 2x_2 + 4x_3 & = & 3 \\ x_1 + 3x_2 + 5x_3 & = & 4 \end{array} \quad (c) \quad \begin{array}{rcl} x_1 + x_2 + 3x_3 & = & 2 \\ x_1 + 2x_2 + 4x_3 & = & 3 \\ x_1 + 3x_2 + 5x_3 & = & 3 \end{array}$$

4. Find all of the eigenvalues and a basis for each of the associated eigenspaces for the given matrix:

$$(a) \quad A = \begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix} \quad (b) \quad A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \quad (c) \quad A = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 3 & 1 \\ 0 & 5 & -1 \end{pmatrix} \quad (d) \quad A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$