1. The exponential distribution is a model for the lifetime of a certain part. Forty such parts are tested. They are not observed continuously but rather are checked every ten hours to see whether they have failed. The test is terminated after 40 hours. The lifetime in hours of the 40 parts as determined by this test are:

<table>
<thead>
<tr>
<th>Interval</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0,10]</td>
<td>14</td>
</tr>
<tr>
<td>(10,20]</td>
<td>8</td>
</tr>
<tr>
<td>(20,30]</td>
<td>5</td>
</tr>
<tr>
<td>(30,40]</td>
<td>4</td>
</tr>
<tr>
<td>(40,Inf]</td>
<td>9</td>
</tr>
</tbody>
</table>

(a) Write the likelihood function for these data (in terms of the unknown parameter $\lambda$).
(b) Find the maximum likelihood estimate of $\lambda$.
(c) Test the null hypothesis that $\lambda = .05$.
(d) Test the hypothesis that the exponential distribution is an appropriate model for these data.

2. Researchers developed a high-energy electron beam process to aid in the hardening of cast iron. They investigated two different factors that had an effect on the success of the process. Factor A is the travel speed in mm/s of the beam and Factor B is the accelerating voltage in volts. The outcome is the “Vickers hardness” of the material which is a certain measure of the ability of the material to resist deformation from a force. The units of Vickers hardness are kilograms-force per mm$^2$. The researchers tested three levels of each factor and had six observations for each treatment. The data are available by

```r
hard <- read.csv("http://www.calvin.edu/~stob/courses/m344/S15/Hardness.csv")
str(hard)
'data.frame': 54 obs. of 3 variables:
$ A : int 10 10 10 10 10 10 10 10 10 10 ... 
$ B : int 10 10 10 10 10 10 25 25 25 25 ... 
$ Hardness: int 875 896 921 686 642 613 712 719 698 621 ...
```

The researchers treated each factor as a categorical variable and did an analysis of variance. They included an interaction term in their model.

(a) Fit the same model that the researchers must have fit. (Remember to change your variables to factors). Give a careful statistical discussion of the relationships among the three variables (factors A and B and Hardness).
(b) The researchers treated the factors as categorical variables rather than as quantitative variables even though they were quantitative in nature. Using the data, give a reason as to why they might have done this.
(c) Refit the model treating the two factors as quantitative variables. Compare the two models in ways that you think are useful for deciding whether to choose one over the other.