Chapel today: *Getting Started*, Student Worship Team

1. Transformations of random variables \(X\). If \(t\) is a function applied to data then \(Y = t(X)\) is the corresponding random variable.

2. If \(Y = t(X)\), we want to find \(E(Y)\) given the pdf of \(X\).

3. Example: \(X\) is the number that appears on a fair die. \(t(x) = |X - 3|\).

\[
\begin{array}{c|cccccc}
 x & 1 & 2 & 3 & 4 & 5 & 6 \\
 f(x) & 1/6 & & & & & \\
 xf(x) & 1/6 & & & & & \\
\end{array}
\]

\[
E(X) = \sum xf(x) =
\]

\[
\begin{array}{c|cccc}
 y & 0 & 1 & 2 & 3 \\
 f(y) & 1/6 & & & \\
 yf(y) & 1/6 & & & \\
\end{array}
\]

\[
E(Y) = \sum yf(y) =
\]

4. Shortcut: \(E(t(X)) = \sum t(x)f(x)\).

5. The same shortcut works for continuous random variables. \(E(t(x)) = \int_{-\infty}^{\infty} t(x)f(x) \, dx\).

6. Some rules (here \(c\) is a constant)

\[
E(cX) = cE(X) \quad E(t(X) + u(X)) = E(t(X)) + E(u(X))
\]

7. The variance of a random variable \(X\) is \(E[(X - \mu)^2]\). The variance of \(X\) is written \(\text{Var}(X)\) or \(\sigma_X^2\). The standard deviation is \(\sigma = \sqrt{\text{Var}(X)}\).

8. Example: The variance of \(X\) where \(X\) is the number of heads in 4 coin tosses.

9. Useful shortcut: \(\text{Var}(X) = E(X^2) - \mu_X^2\).

10. The variance of our favorite random variables:

    (a) If \(X \sim \text{Binom}(n, \pi)\) then \(\text{Var}(X) = np(1 - \pi)\).

    (b) If \(X \sim \text{Hyper}(m, n, k)\) then \(\text{Var}(X) = k \left( \frac{m}{m+n} \right) \left( \frac{n}{m+n} \right) \left( \frac{m+n-k}{m+n-1} \right)\).

    (c) If \(X \sim \text{Unif}(a, b)\) then \(\text{Var}(X) = (b-a)^2/12\).

    (d) If \(X \sim \text{Exp}(\lambda)\) then \(\text{Var}(X) = 1/\lambda^2\).
Homework

1. Note: the homework due today will be collected on Thursday rather than Tuesday.

2. Read Section 4.6.


4. Additional problems:

   (a) Let $X$ be the the random variable that results from adding the numbers on two fair dice. Compute the mean and variance of $X$.

   (b) Refer to Example 3.4.2 of the text. Suppose that insurance company sells a five year term policy worth $100,000 to a 55 year-old male for a lump sum payment of $5,000. Let $X$ be the value of this policy to the company in 5 years. Note that $X$ has two possible values, $5,000 if the policy holder lives and $-95,000 if the policy holder dies. Using the data in the table to estimate the probability of death in 5 years, compute the expected value and the variance of this random variable $X$. (There are two complications that we are omitting from consideration here. First, there is a time value of money. $5,000 up front is worth more than $5,000 in five years. Second, there is the fact that insurance premiums are usually paid over time, typically in six month installments.)

   (c) For every $k$, the function $f(x) = (k + 1)x^k$ is the density of a random variable $X$ that has values $0 \leq x \leq 1$. Compute the mean and variance of this random variable for each $k$. Evaluate your expression for $k = 1, 5, 10$. 