P(A)   Probability of A
P(B)   Probability of B
P(\bar{A})   Probability that A doesn’t happen
P(A \mid B)   Probability of A given B
P(B \mid A)   Probability of B given A
P(A \cap B)   Probability of A and B

For each of the following random processes, make estimates of the indicated probabilities. For some, you will have a model in mind for which there is really only one right answer. For other of these, you do not have enough data to make a perfect model so you will have to provide your own estimate. However the answers need to be internally consistent.

**Experiment:** A red die and a green die are tossed

A    red die comes up 2
B    green die comes up 3

P(A)   
P(B)   
P(A \mid B)   
P(B \mid A)   

**Experiment:** The Packers play the Forty-Niners Sunday.

A    The Packers win.
B    The Forty-Niners win.

P(A)   
P(B)   
P(A \mid B)   
P(B \mid A)   

**Experiment:** A Calvin student is chosen “at random”.

A    the student is male
B    the student is an engineer

P(A)   
P(B)   
P(A \mid B)   
P(B \mid A)   

**Experiment:** Two cards are taken from the top of a well-shuffled deck of cards.

A    The first card is a heart
B    The second card is a heart

P(A)   
P(B)   
P(A \mid B)   
P(B \mid A)   

\[ P(A \cap B) = P(A) P(B \mid A) = P(B) P(A \mid B) \]

Using your estimates from the previous page and the multiplication rule above, compute the probability that:

The red die is 2 and green die is 3

A randomly chosen Calvin student is a male engineer

Both the Packers and the Forty-Niners win Sunday

The top two cards in a deck of cards are red

A probability tree helps organize the various probabilities involved with a random experiment.

For example, consider the experiment of choosing two cards from a well-shuffled deck. Let \( A \) be the event that the first card is a red card and \( B \) the event that the second card is a red card.

So the probability of two reds is \( \frac{25}{102} \) but the probability of one red and one black is \( \frac{52}{102} \) (\( \frac{26}{102} + \frac{26}{102} \)).
Carissa Verkaik makes about 86% of her free-throws. Suppose she is shooting two free-throws. Draw a probability
tree and write in the probabilities. You need to make an assumption about her throws. (What is it?)

What is the probability that Carissa makes both free throws? ______________

What is the probability that she makes neither free throw? ______________

What is the probability that she makes exactly one of them? ______________

Trees can help us think about more complicated experiments. What do the trees look like in the following situations? (Sketch the shape of the tree but you need not label the branches or nodes.)

• A red and a green die are tossed and the number that appears on each is recorded.

• Three cards are chosen and their color recorded.
As we saw above, we often use $P(A \mid B)$ to help us compute $P(A \cap B)$. However the multiplication equation can be used just as easily to compute $P(A \mid B)$ from $P(A \cap B)$.

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

In this activity, we investigate the issue of universal testing for a serious disease such as AIDS. There is a relatively inexpensive test that can be administered in any doctor’s office. Here are some facts about HIV and this test.

- About 469 of every 100,000 adults living in the United States are HIV positive. (0.5%)
- If a person is HIV positive, this test will detect this 98.5% of the time (and be wrong 1.5% of the time).
- If a person does not have the virus, the test will correctly indicate this 99.7% of the time (and be wrong .3% of them time).

Consider the experiment of choosing a person at random from the adult population and administering the test. Consider the two events:

- $H$ the person is HIV positive
- $T$ the test says that the person is HIV positive

The above facts suggest that we can compute probabilities on the following tree:

```
          T       H ∩ T
          /       /
         H   H ∩ T
        /   /      
       T   H ∩ T
      /   /      
     H   T      H ∩ T
```

We know $P(T \mid H)$. But what a patient really wants to know is $P(H \mid T)$. Compute it.

$$P(H \mid T) = \frac{P(H \cap T)}{P(T)} =$$

From the last two bullet points above, most people would say that the tests are over 99% accurate. Explain why this is a misleading claim.
The diagram below represents a production process for aluminum cans. Coils of aluminum are first “cupped” (rolled into cylinders closed at the bottom, open at the top), “washed”, and then sent to the “printer” for labeling. There are several printers and any one can be used to print the cans coming from washing. The cans are then “filled” (and capped). There are two fill machines. One accepts the cans on the pallets that are delivered from the print operation and the other accepts cell bins that hold approximately 100K cans. The cans sent to this latter machine need to be depalletized and put into these bins before filling.

It is desired to estimate the probability that this production run will function for one day without failing. From the above diagram, for example, the run fails if cupping fails but can continue to run if one or even two printers fail. From past experience with the machines, the estimates that each machine functions for a day without failing are: $P(A) = 0.995$, $P(B) = 0.99$, $Prob(C) = P(D) = P(E) = 0.95$, $P(F) = 0.90$, $P(G) = 0.90$, $P(H) = 0.98$. We can also assume that the components function (and fail) independently.

What is the probability that the production run functions for a day without failing?