Suppose $X_1, \ldots, X_k$ are independent random variables with means $\mu_{X_i}$ and standard deviations $\sigma_{X_i}$. Then mean and standard deviations of the random variable $Y = f(X_1, \ldots, X_k)$ are approximated by the following

$$\mu_Y \approx f(\mu_{X_1}, \ldots, \mu_{X_k})$$

and

$$\sigma_Y \approx \sqrt{\left( \frac{\partial f}{\partial X_1} \right)^2 \sigma_{X_1}^2 + \cdots + \left( \frac{\partial f}{\partial X_k} \right)^2 \sigma_{X_k}^2}$$

Here the partial derivatives are evaluated at the point $(\mu_{X_1}, \ldots, \mu_{X_k})$.

### Dimes

On the first day we tried to estimate the total number of dimes in a sack by using a sample of 30 dimes.

Our estimate of $N$, the number of dimes in the sack, was

$$N = \frac{T}{\bar{d}}$$

where $T$ is the total weight of dimes in the sack and $\bar{d}$ is the sample mean of our sample of 30 dimes. Here are the measurements

$$T = 10.4 \text{kg} \quad \bar{d} = 2.258 \text{gm}$$

1. What is the estimate for the number of dimes in the sack based on these measurements? (It might be wise to convert the measurements so that they have the same unit!)

2. Write the propagation of uncertainty formula for $\sigma_N$ in terms of $\sigma_T$ and $\sigma_{\bar{d}}$.

3. To get an estimate of $\sigma_{\bar{d}}$, we use a type A estimate since we made 30 repeated measures. What is our estimate of $\sigma_{\bar{d}}$? (Hint: we can get this from `lm`).

4. To get an estimate of $\sigma_T$ we need a type B estimate since we only weighed the whole collection of dimes once. Suppose we know that we used a digital scale with readings reported to 0.1 kg. Use a type B estimate with a uniform model to compute a standard uncertainty for $T$.

5. Use the uncertainty estimates and the propagation of uncertainty formula to compute an estimate of $\sigma_N$.

6. Now do a simulation. Namely suppose a uniform distribution for $T$ and a normal distribution for $\bar{d}$ and compute the mean and standard deviation of 1,000 trials of this experiment. (We can simulate numbers from a uniform distribution using `runif()`) How does the standard deviation of your simulated values compare with your estimate of $\sigma_N$ from the propagation of uncertainty formula?
Resistance in parallel

A resistor with color labels “brown-black-red-gold” resistor is one that is $1,000 \pm 5\%$ ohms. The resistance tolerance (5% or 50 ohms here) is generally taken to be a guarantee of the tolerance. Two such resistors in a circuit in parallel have resistant $R$ given by the following:

$$R = \frac{R_1 R_2}{R_1 + R_2}$$

1. The uncertainty in the individual resistors, 50 ohms, is a “worst case” scenario. What model could we use to estimate the standard uncertainty?
2. Use your model to report a standard uncertainty.
3. Carefully compute the partial derivatives in the propagation of uncertainty formula.
4. Compute the standard uncertainty in $R$ from the propagation of uncertainty formula.

SAT Scores

If $X$ and $Y$ are not independent and $Z = X + Y$ it is still true that $\mu_Z = \mu_X + \mu_Y$ but the rule for combining standard deviations is no longer true. Suppose that we choose a random Calvin senior and observe the SAT scores of that senior. (Many Calvin seniors do not have SAT scores so we only consider those who do.) We let $X$ be the verbal SAT score of the random student and $Y$ be the math SAT.

1. SAT mathematics and verbal scores are not independent – what do you think the relationship between $X$ and $Y$ is and how strong do you think it is?
2. We will investigate a particular population – the 2004 senior class at Calvin. Use these data to give a numerical description of the strength of this relationship.
3. In this example, we do not need to approximate the standard deviations since we have the whole population. Compute the population mean and standard deviation for SATV and SATM for this group of seniors. Complete the following table.

<table>
<thead>
<tr>
<th></th>
<th>$\mu$</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SATM</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SATV</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4. From the standard deviations above, what would the standard deviation of SATV+SATM be if the verbal and math scores were independent?
5. Find the mean and standard deviation of SATV+SATM for this population.