MATH 365: Homework #35
Assigned: Mon., May 7, 2012
Due Date: Wed., May 9, 2012

Read/skim Section 9.1 of the text. It is long, and you do not need to understand all of the examples. One thing to note is that, generally speaking, the sequences listed in Table 9.1 (p. 342) are taken to be causal. That is, Entry 12 is for the sequence

\[ x[n] = \begin{cases} 
\cos(an), & n \geq 0, \\
0, & n < 0.
\end{cases} \]

\[ \star \] \(50\) Entry 5 in Table 9.1 (p. 342) gives that, for fixed \(a \in \mathbb{C}\), the causal sequence

\[ \begin{cases} 
e^an, & n \geq 0 \\
0, & n < 0
\end{cases} \]

has \(z\)-transform \( \frac{z}{z - e^a} \).

Use this entry to show that the causal sequence

\[ \begin{cases} \sin(an), & n \geq 0 \\
0, & n < 0
\end{cases} \]

has \(z\)-transform \( \frac{z \sin a}{1 + z^2 - 2z \cos a} \).

\[ \star \] \(51\) Find inverse \(z\)-transforms for the following functions in the indicated regions of convergence.

(a) \( X(z) = \frac{1}{1 + 1/(3z)}, \ |z| < \frac{1}{3} \)

(b) \( X(z) = \frac{1}{1 + 1/(3z)}, \ |z| > \frac{1}{3} \)

(c) \( X(z) = \frac{z^4}{z + 2}, \ |z| < 2 \)

(d) \( X(z) = \frac{z^4}{z + 2}, \ |z| > 2 \)

9.1.8(a) Find the inverse \(z\)-transform of

\[ X(z) = \frac{z^2}{z^2 - 4z + 3} = \frac{z^2}{(z - 1)(z - 3)} = \frac{1}{(1 - z^{-1})(1 - 3z^{-1})} \]

via two methods:

(i) partial fractions and Table 9.1, and

(ii) using residues.

\[ \star \] \(52\) In class we showed that, if \(X(z)\) is the \(z\)-transform of a sequence \(x = (x[n])_{n \in \mathbb{Z}}\) and we define a sequence \(w\) to be a shifted version of the sequence \(x\)—that is, \(w[n] = x[n - n_0]\) for fixed \(n_0 \in \mathbb{Z}\)—then the \(z\)-transform of \(w\) is given by

\[ W(z) = z^{-n_0}X(z). \]
Suppose we restrict ourselves to causal sequences. Not only does this mean the original sequence \( x[n] = 0 \) for all \( n < 0 \), but, when we shift such a sequence, anything that would correspond to a negative index \( n \) is lost, so

\[
w[n] = \begin{cases} 
  x[n - n_0], & n \geq 0, \\
  0, & n < 0.
\end{cases}
\]

Show that, under this more restrictive type of shift, Equation (1) continues to hold in the case \( n_0 \geq 0 \); but, for the case \( n_0 < 0 \), we have the modified formula:

\[
W(z) = z^{-n_0} \left( X(z) - \sum_{k=0}^{-(n_0+1)} x[k]z^{-k} \right).
\]

\( \star 53 \) Assume that \( a = (a[n])_{n \in \mathbb{Z}} \) is a causal sequence satisfying the difference equation

\[
a[n + 2] - 5a[n + 1] + 6a[n] = u[n],
\]

with initial conditions \( a[0] = 2, a[1] = 3 \). Use the result of the previous problem to find an explicit formula for \( A(z) \), the \( z \)-transform of the sequence \( a \).

Optionally, you may also invert \( A(z) \) to get an explicit formula for \( a[n], n \in \mathbb{Z} \).