MATH 365: Homework #21
Due Date: Fri., Mar. 16, 2012

The problems here are from material covered in Sections 6.1, 6.2 and 6.4. As of yet, we have not covered the material in 6.3.

Do these Exercises not for handing in.

Section 6.1: 1, 2
Section 6.2: 1, 2, 7 parts (a) and (b), 8
Section 6.4: 1, 2, 3, 8

6.1.3 Show that \( \int_{0}^{\infty} e^{-zt} \, dt = \frac{1}{z} \), provided \( \text{Re}(z) > 0 \).

6.2.6 Evaluate \( \int_{C} y \, dz \) for \((-i)\) to \(i\) along the following contours.

(a) The polygonal path \( C \) with vertices \((-i), (-1 - i), (-1), i\).
(b) The half circle (left half of \( C_{1}(0) \)) oriented clockwise.

6.2.10a Show that \( \left| \int_{\gamma} \frac{dz}{z^{2} - 1} \right| \leq \frac{\pi}{3} \), where \( \gamma \) is the first quadrant portion of \( C_{2}(0) \), oriented positively.

6.4.16 (a) Let \( z_{1}, z_{2} \) be points in the right half-plane \( \{ x + iy \mid x > 0 \} \), and let \( \gamma \) be a curve in the right half-plane from \( z_{1} \) to \( z_{2} \). Show that \( \int_{\gamma} \frac{dz}{z^{2}} = \frac{1}{z_{1}} - \frac{1}{z_{2}} \).

(b) What happens if we remove the restriction of staying in the right half-plane? What requirement(s) must you impose so that the integral expression continues to hold?