Find a formula for the complex constant ("phasor") $\alpha$ so that the general sinusoid at frequency $\nu = \omega / (2\pi)$,

$$a_1 \cos(\omega t + \gamma_1) + a_2 \cos(\omega t + \gamma_2) + \cdots + a_m \cos(\omega t + \gamma_m) + b_1 \cos(\omega t + \delta_1) + b_2 \cos(\omega t + \delta_2) + \cdots + b_n \cos(\omega t + \delta_n)$$

is expressed as $\text{Re} \, \alpha e^{i\omega t}$.

Using techniques demonstrated in class, find the steady-state current output $I_s$ of the circuits pictured below.

Note: For the circuit we did in class (the one with a resistor and capacitor in parallel, running into an inductor), we get the expression

$$I_s = \text{Re} \left( \frac{e^{i\omega t}}{R_{\text{eff}}} \right) = \frac{R \cos(\omega t) - [R^2 \omega C (1 - \omega^2 LC) - \omega L] \sin(\omega t)}{R^2 (1 - \omega^2 LC)^2 + \omega^2 L^2},$$

under the assumption that $V_s = \cos(\omega t)$. Make this assumption about $V_s$ in parts (a)–(c) as well.

Consider the curves

$$z_1(t) = t + it^2, \quad -1 \leq t \leq 1,$$

and

$$z_2(t) = 1 - 2t^2 + i(1 - 2t^2)^2, \quad -1 \leq t \leq 1.$$
(a) The two curves have the same range. Sketch it. If you wanted a function 
\( y = f(x), a \leq x \leq b, \) whose graph (in \( \mathbb{R}^2 \)) was the same as the range of these curves, what function \( f \) and endpoints \( a, b \) to its domain would do the job?

(b) In class we discussed the integral formula for arc length. For all but a few specialized cases, however, it is difficult to antidifferentiate \(|z'(t)|\), making it more practical to approximate the arc length using a numerical integration method like Simpson’s rule (this link provides a refresher) than to evaluate it via the Fundamental Theorem of Calculus. Use the applet at http://www.math.ucla.edu/~ronmiech/Java_Applets/Riemann/index.html to find approximate arc lengths associated with the curves \( z_1(t), z_2(t) \). They will not be equal. Explain why this is so.

(c) Show that \( z_1 \) is a smooth curve, but \( z_2 \) is not.