Solutions to PS #29

$\star$44. **Nintegrate** seems to work better, at least for the integrals of the simple functions, than **Integrate**. The relevant syntax is something like

NIntegrate[s[2,x], {x,-2,2}]

which produces an answer despite also raising complaints that the integral is difficult to calculate numerically.

$\star$45. Let $P = \{x_0, x_1, \ldots, x_n\}$ with $x_{j-1} < x_j$ for $j = 1, \ldots, n$ be a partition of $[0, 1]$. Since each subinterval $I_j := [x_{j-1}, x_j]$ contains both a rational and an irrational, we have

$$\inf_{t \in I_j} \chi_A(t) \;=\; 0 \qquad \text{and} \qquad \sup_{t \in I_j} \chi_A(t) \;=\; 1 \,,$$

which means

$$L(\chi_A, P) \;:=\; \sum_{j=1}^{n} \left(\inf_{t \in I_j} \chi_A(t) \right) (x_j - x_{j-1}) \;=\; 0 \,,$$

and

$$U(\chi_A, P) \;:=\; \sum_{j=1}^{n} \left(\sup_{t \in I_j} \chi_A(t) \right) (x_j - x_{j-1}) \;=\; \sum_{j=1}^{n} (x_j - x_{j-1}) \;=\; 1 \,.$$

Thus, the lower Riemann integral of χ_A is

$$\underline{\int_0^1} \chi_A(t)\, dt \;:=\; \inf \{ L(\chi_A, P) \,|\, P \text{ a partition of } [0,1] \} \;=\; 0 \,,$$

while the upper Riemann integral is

$$\overline{\int_0^1} \chi_A(t)\, dt \;:=\; \sup \{ U(\chi_A, P) \,|\, P \text{ a partition of } [0,1] \} \;=\; 1 \,.$$

Since these lower and upper integrals are not equal, there is no common value which we may define to be the Riemann integral of χ_A over $[0, 1]$.