Solutions to PS #13

5.17 By Taylor's Thm. $\exists\, t \in (-1, 0)$ s.t.

$$f(-1) \;=\; \frac{f(0)}{0!}\,(-1)^0 + \frac{f'(0)}{1!}\,(-1)^1 + \frac{f''(0)}{2!}\,(-1)^2 + \frac{f'''(t)}{3!}\,(-1)^3.$$

Using the values supplied, this expression becomes

$$f^{(3)}(t) \;=\; 3f''(0). \tag{1}$$

Another application of Taylor's Thm., this time on the right side of $x = 0$, gives that $\exists s \in (0, 1)$ s.t.

$$f(1) \;=\; \frac{f(0)}{0!}\,(1)^0 + \frac{f'(0)}{1!}\,(1)^1 + \frac{f''(0)}{2!}\,(1)^2 + \frac{f'''(s)}{3!}\,(1)^3.$$

Again using the values supplied, we have

$$-f^{(3)}(s) \;=\; 3f''(0) - 6. \tag{2}$$

Subtracting equation (2) from (1), we have

$$f^{(3)}(t) + f^{(3)}(s) \;=\; 6,$$

which means that at least one of the numbers $f^{(3)}(t)$, $f^{(3)}(s)$ is ≥ 3.