MATH 362: Problem Set #20

Read pp. 300-307 of Chapter 11, up to (but not including) the proof of Theorem 11.10.

⋆30. Prove that rectangles in \( \mathbb{R}^2 \) have content (as defined by Jordan), and that the content \( \mu(R) \) of a rectangle \( R \) agrees with \( m(R) \).

Hint: Show individually that \( m(R) = \overline{\mu}(R) \) and \( m(R) = \underline{\mu}(R) \).

⋆31. In the proof of Theorem L.6 (iv), we produced a closed set \( F \) inside \( A \), and used its compactness to get that

\[
\mu(F) \leq \mu(A_1) + \mu(A_2) + \cdots + \mu(A_N).
\]

Explain why this was necessary—that is, why we bothered with compactness at all, and did not simply use the fact that \( F \) is covered by the countable collection \((A_n)\) to jump directly to

\[
\mu(F) \leq \sum_{n=1}^{\infty} A_n.
\]

⋆32. Let \( E \) denote the set of rational points in the interval \([a,b]\). Prove that the lower (Jordan) content of \( E \) is 0, while the upper content of \( E \) is at least \((b-a)\).