MATH 362: Problem Set #20

Read pp. 300-307 of Chapter 11, up to (but not including) the proof of Theorem 11.10.

$\star$30. Prove that rectangles in $\mathbb{R}^2$ have *content* (as defined by Jordan), and that the content $\mu(R)$ of a rectangle R *agrees* with $m(R)$.

Hint: Show individually that $m(R) = \overline{\mu}(R)$ and $m(R) = \underline{\mu}(R)$.

$\star$31. In the proof of Theorem L.6 (iv), we produced a closed set F inside A, and used its compactness to get that

$$\mu(F) \;\leq\; \mu(A_1) + \mu(A_2) + \cdots + \mu(A_N)\ .$$

Explain why this was necessary—that is, why we bothered with compactness at all, and did not simply use the fact that F is covered by the countable collection (A_n) to jump directly to

$$\mu(F) \;\leq\; \sum_{n=1}^{\infty} A_n\ .$$

$\star$32. Let E denote the set of rational points in the interval $[a, b]$. Prove that the lower (Jordan) content of E is 0, while the upper content of E is at least $(b - a)$.