MATH 362: Problem Set #8

3.4 Find the upper and lower limits (the lim inf and lim sup) of the sequence (s_n) defined by

$$s_1 \;=\; 0\,; \qquad s_{2m} \;=\; \frac{s_{2m-1}}{2}\,; \qquad s_{2m+1} \;=\; \frac{1}{2} + s_{2m}\,.$$

$\star$1. Theorem S.9 says: If (s_n) is a real sequence, then $\exists$ a subsequence (s_{n_k}) s.t. $s_{n_k} \to \limsup s_n$. Similarly, there is a subsequence converging to $\liminf s_n$.

The proof given in class addressed the cases where $\liminf s_n$, $\limsup s_n \in \mathbb{R}$. Show that S.9 holds when $\liminf s_n$, $\limsup s_n = \pm\infty$.