Solutions to PS #2

2. (a) $1 \times 2^4 + 1 \times 2^3 + 1 \times 2^1 + 1 \times 2^0 + 1 \times 2^{-2} = 27.25$

   (b) $2 \times 3^2 + 1 \times 3^0 + 1 \times 3^{-1} + 1 \times 3^{-2} + 2 \times 3^{-3} \doteq 19.\overline{518}$

   (c) $5 \times 16^2 + 6 \times 16 + 12 \times 16^0 + 15 \times 16^{-1} = 1388.9375$

3. (a) Using long division, we have

$$\begin{aligned} 38.90625 - 2^5 &= 6.90625 \\ 6.90625 - 2^2 &= 2.90625 \\ 2.90625 - 2^1 &= 0.90625 \end{aligned} \qquad \begin{aligned} 0.90625 - 2^{-1} &= 0.40625 \\ 0.40625 - 2^{-2} &= 0.15625 \\ 0.15625 - 2^{-3} &= 0.03125 \\ 0.03125 &= 2^{-5} \end{aligned}$$

   Thus, $(38.90625)_{10} = (100110.11101)_2$.

   (b) The easiest way is to note that $16 = 2^4$, and so each hexadecimal digit corresponds to 4 binary digits. To be more explicit,

hex	4-bit binary	hex	4-bit binary	hex	4-bit binary	hex	4-bit binary
0	0000	4	0100	8	1000	C	1100
1	0001	5	0101	9	1001	D	1101
2	0010	6	0110	A	1010	E	1110
3	0011	7	0111	B	1011	F	1111

   Thus, $(56C.F)_{16} = (010101101100.1111)_2$.

4. (a) Here $s = -1$, $e = (11010011)_2 - (127)_{10} = (84)_{10}$, and $m = 1 + (0.00101110111100101000000)_2 = (1.183387756347656)_{10}$. Our number, then, is

$$s * m * 10^e = -1.183387756347656 \times 2^{84} = -2.28900482122321 \times 10^{25} .$$

   (b) This time $s = 1$, $e = (00011111)_2 - (127)_{10} = (-96)_{10}$, and $m = 1 + (0.1001101)_2 = (1.6015625)_{10}$. So, the number is

$$s * m * 10^e = 1.6015625 \times 2^{-96} = 2.02145606962884 \times 10^{-29} .$$

5. (a) Using long division, we have

$$\begin{aligned} 0.1 - 2^{-4} &= 0.0375 \\ 0.0375 - 2^{-5} &= 0.00625 \\ 0.00625 - 2^{-8} &= 0.00234375 \end{aligned} \qquad \begin{aligned} 0.00234375 - 2^{-9} &= 0.000390625 \\ 0.000390625 - 2^{-12} &= 0.000146484375 \\ 0.000146484375 - 2^{-13} &= 0.0000244140625 \end{aligned}$$

Thus far, we have binary representation $(0.0001100110011\ldots)_2$. We might suspect that this is a repeating representation, continuing with a neverending sequence of two zeros followed by two ones. The string 0011 would correspond to a reduction by 4 in the power of 2's. Thus, we can determine if, indeed, things are repeating by checking things like

$$\begin{aligned} 2^4 \times 0.00234375 &= 0.0375 & \qquad 2^4 \times 0.000146484375 &= 0.00234375 \\ 2^4 \times 0.00390625 &= 0.00625 & \qquad 2^4 \times 0.0000244140625 &= 0.000390625\,. \end{aligned}$$

Thus, $(0.1)_{10} = (0.000\overline{1100})_2$.

(b) Since $1/10 > 0$, our sign bit is 0. Carrying our binary representation out just so far as we need, we have

$$0.000\overline{1100} = 1.1001\ \ 1001\ \ 1001\ \ 1001\ \ 1001\ \ 1001\ \ \ldots\ \ \times 2^{-4}\,,$$

showing that the 24th bit is a 1. We also see that $e = -4$, so our 8 bits devoted to exponent will correspond to the binary representation of $(-4) + 127 = 123$. Thus, we have IEEE 32-bit representation

$$0\ \ 01111011\ \ 10011001100110011001101\,.$$

(c) Converting the representation of part (b) back to a decimal number yields $(0.100000001490116)_{10}$. Evidently, $1/10$ is not a machine number, as it was the nearest machine number to which it was converted. This fact actually proved fatal to 28 American soldiers in 1991 during the Gulf War. Read about it at http://www.ima.umn.edu/~arnold/455.f97/notes.html.