MATH 335: Numerical Analysis

Problem Set 21, Final version

Due Date: Mon., May 11, 2009

Read Sections 12.1–12.2 from Kharab & Guenther, along with your in-class notes.

CP 12.1.4 Solve the BVP

$$x^2y'' - xy' + y = \ln x\,, \quad x \in (1,2)\,, \qquad \text{subject to} \qquad y(1) = 0\,, \quad y(2) = -2\,,$$

using the finite difference method with $n = 8$.

⋆44 You may download my dirichletShooter.m and rk4sys.m routines (the latter has been modified for compatability's sake, so you should download it again even if you already had a copy), and use them to solve the following problems.

(a) **Ex. 12.2.4**: Find the approximate solution of the BVP

$$y'' = y' - \sin(xy)\,, \quad x \in (0,1)\,, \qquad \text{subject to} \qquad y(0) = 1\,, \quad y(1) = 1.5\,.$$

Use the stepsize $h = 0.1$.

(b) **Ex. 12.2.5**: Find the approximate solution of the BVP

$$y'' = y\cos^2 x - e^{\sin x}\sin x\,, \quad x \in (0,\pi)\,, \qquad \text{subject to} \qquad y(0) = 1\,, \quad y(\pi) = 1\,,$$

using 20 subintervals. Compare with the exact solution $y = e^{\sin x}$.

Challenge: I claimed my method for solving the linear BVP

$$y'' + p(x)y' + q(x)y = r(x)\,, \qquad y(a) = \alpha, \quad y(b) = \beta\,,$$

which involved solving the two IVPs

$$\begin{aligned} u'' + p(x)u' + q(x)u &= r(x)\,, \qquad u(a) = \alpha, \quad u'(a) = 0\,,\\ v'' + p(x)v' + q(x)v &= r(x)\,, \qquad v(a) = \alpha, \quad v'(a) = 1\,,\end{aligned}$$

and then combining them via $y(x) = \lambda u(x) + (1-\lambda)v(x)$ (with $\lambda = (\beta - v(b))/(u(b) - v(b))$) was *better* than the book's method, which solves the two problems

$$\begin{aligned} u'' + p(x)u' + q(x)u &= r(x)\,, \qquad u(a) = \alpha, \quad u'(a) = 0\,,\\ v'' + p(x)v' + q(x)v &= 0\,, \qquad v(a) = 0, \quad v'(a) = 1\,,\end{aligned}$$

and then combines them via $y(x) = u(x) + (\beta - u(b))v(x)/v(b)$. What reasons would I give for that claim?